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Reply to ‘Comment on “Breakdown of Hydrodynamics in the Classical 1-D Heisenberg Model,'” by Bohm et al

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de Alcantara Bonfim and Reiter Reply: Böhm, Gerling, and Leschke [1] pointed out an interesting property of the correlation function $C(q,t)$ for the 1D classical Heisenberg model in the limit $q \rightarrow 0$. By reanalyzing previously published data [2], they have found that the second derivative of $C(q,t)$ with respect to q at $q=0$, namely,

$$-\partial^2 C(q,t)/\partial q^2 \equiv \sum_{r=-\infty}^{\infty} r^2 C_r(t), \qquad (1)$$

is well fitted by the relation $0.76Jt\ln(Jt)$, for times $1 \leq Jt \leq 160$, where J is the exchange constant. The fit was done by calculating the right-hand side of (1) using the data for the pair correlation function $C_r(t)$ for values of r up to 100. This result implies that the correlation function $C(q,t)$ may be written as $C(q,t)=\exp[-\text{const} \times A(q)t\ln(t)]$ with $A(q) \rightarrow q^2$ for $q \rightarrow 0$. They argue correctly that the form $A(q)=q^{2.12}$ used in our paper [3] leads to a null result for (1) in contradiction with the numerically fitted form $0.76Jt\ln(Jt)$. To eliminate this contradiction the authors of Ref. [1] proposed a quadratic form for $A(q)$ with a quartic correction, that is, $C(q,t)=\exp[-0.38q^2(1+10q^2)Jt\ln(Jt)]$. They claim that this form fits their data "nearly as well as" the form $C(q,t)=\exp[-0.543q^{2.12}Jt\ln(Jt)]$ found in our paper. We have used our data to plot in Fig. 1 both expressions for $C(q,t)$. It clearly shows that the form proposed in [1] does not fit the data for values of $q \geq \pi/200$. In fact the same q dependence proposed in [1] was used in our preliminary calculations and subsequently discarded for giving poor results compared with the simple power law $q^{2.12}$. For values of $q < 3\pi/200$ the effect of the q^4 term is very small ($<2\%$) showing that conventional q dependence does not provide an adequate form for the correlation function $C(q,t)$. Furthermore, the difference does not lie in any disagreement about the data. At the lowest value of q measured, the coefficient of q^2 in the expression $0.38q^2$ and in $0.543q^{2.12}$ is nearly identical (0.38 and 0.33, respectively).

To further understand the problem with the q dependence of $C(q,t)$ we have performed extensive spin-dynamics simulations on the 1D Heisenberg model with random exchange. We found that the correlation function behaves as $C_r(q,t)=\exp(-0.665q^2t)$ for the same range of q values used in the case of uniform exchange with no higher-order correction in q being necessary.

To resolve the contradiction that $A(q) \rightarrow q^2$ for $q \rightarrow 0$ and the fact that the scaling form for $C(q,t)$ is observed only if $A(q)=q^{2.12}$ we propose that there is a crossover for $A(q)$ from q^2 to $q^{2.12}$ for values of q somewhere in the interval $(0,\pi/200)$. The position of the crossover is presumably time dependent, as we can see no mechanism for the introduction of a fixed length scale. Then the functional form would change over at some value of $q^2t\ln(t)$ so that

$$\lim_{t\rightarrow\infty}\lim_{q\rightarrow 0} C(q,t) \neq \lim_{q\rightarrow 0}\lim_{t\rightarrow\infty} C(q,t).$$

We should point out that the true form of $A(q)$ is unknown and the form $q^{2.12}$ is an effective representation of $A(q)$ for q in the interval $(\pi/200, 5\pi/200)$.

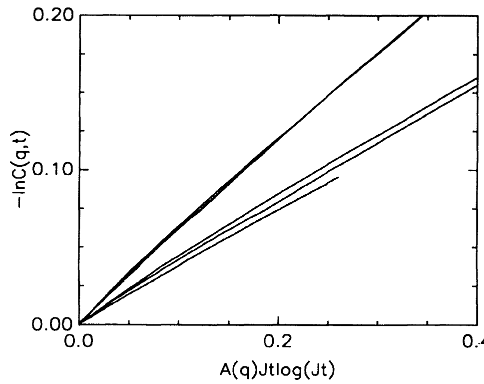

FIG. 1. The logarithm of the spin correlation function $C(q,t)$ for the 1D classical Heisenberg model exchange at infinite temperature plotted against $A(q)t\ln(t)$ for three values of q ranging from $\pi/200$ to $3\pi/200$. The lower set of lines correspond to $A(q)=q^2(1+10q^2)$ and the upper set to $A(q)=q^{2.12}$. The lines represent the simulation done in a lattice with 400 spins averaged over 15000 random initial conditions.

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[2] R. W. Gerling and D. P. Landau, Phys. Rev. B **42**, 8214 (1990).
[3] O. F. de Alcantara Bonfim and G. Reiter, Phys. Rev. Lett. **69**, 367 (1992).