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Comment on “Inelastic Collapse of a Randomly Forced Particle”

In their Letter [1] Cornell et al. investigated the dynamics of a particle in a one-dimensional box of length $l$ subject to a random force $\eta(t)$. The particle motion is governed by the equation

$$\frac{d^2x}{dt^2} = \eta(t),$$

where $\eta(t)$ is a Gaussian noise with zero mean and a delta-correlated time dependence $\langle \eta(t)\eta(t') \rangle = 2\delta(t-t')$. The collisions with the boundaries are inelastic, with coefficient of restitution $r$; that is, whenever the particle collides with a wall it rebounds with a speed which is $r$ times smaller than the speed before collision.

The authors report a remarkable transition in which below a critical value, $r_c = \exp(-\pi/\sqrt{3})$, the particle adheres to one of the walls after colliding with it an infinite number of times in a finite time. For $r > r_c$ the motion is ergodic. They also find that the value of $r_c$ is independent of the box size and of the magnitude of a viscous damping term added to the equation of motion, Eq. (1).

A series of rescaling of the position and time variables and followed by a transformation onto a stationary Gaussian process, the equation of motion is mapped onto that of a Brownian particle subject to a harmonic force, Eq. (6) of [1]. The critical value $r_c$ was obtained from a mean-field analysis. They also performed a numerical integration of Eq. (6) and, upon transforming back onto the particle’s original coordinates, they found evidence for the inelastic collapse for the particle trajectories, as predicted by their mean-field calculation. Numerical results for the collapse transition were displayed in Fig. 2 of their Letter.

We decided to numerically integrate Eq. (1) directly, in the hope to reproduce the results of Cornell et al. To our surprise we did not find any instance of a collapse transition for any values of $r$. In order to assert the robustness of the solutions, we used both second- and third-order stochastic Runge-Kutta methods [2] to integrate Eq. (1) for a variety of initial conditions on the positions and velocities. We used integration time steps in the interval $10^{-2}$ to $10^{-4}$. We tested our numerical integration method against the exact result for both the mean square displacement and the mean square velocity of a Brownian particle. The agreement was within 2% or less of the exact values for $10^4$ samples and times up to 150. In Fig. 1 we show our results for the trajectories of a particle with position $0 \leq x \leq 1$ and times up to $t = 100$ (same range of parameters used in Fig. 2 of [1]), for two values of the coefficient of restitution below and above the expected critical value $r_c = 0.163$. Clearly, no collapse transition is observed. We also find that the presence of a viscous damping term in Eq. (1) does not induce a collapse transition for any values of $r$ and coefficient of friction.

We further analyze the model by calculating the power spectrum of $x(t)$. Our analysis shows that the average power spectrum is independent of the coefficient of restitution $r$. In addition, we find that the system exhibits two distinct $1/f^a$ behaviors. For high frequencies we obtain a $1/f^2$ behavior for the power spectrum, as one would expect, which reflects the Brownian character of the particle motion at short times. On the other hand, at low frequencies we find a $1/f^b$ behavior for the power spectrum which can be attributed to the multiple collisions of the particle with the walls. We also find that in the low frequency regime the power $a$ decreases as the damping coefficient increases. Details of these results will be presented elsewhere.

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