2004

Decoherence, the measurement problem, and interpretations of quantum mechanics

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Environment-induced decoherence and superselection have been a subject of intensive research over the past two decades, yet their implications for the foundational problems of quantum mechanics, most notably the quantum measurement problem, have remained a matter of great controversy. This paper is intended to clarify key features of the decoherence program, including its more recent results, and to investigate their application and consequences in the context of the main interpretive approaches of quantum mechanics.

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I. INTRODUCTION

The implications of the decoherence program for the foundations of quantum mechanics have been the subject of an ongoing debate since the first precise formulation of the program in the early 1980s. The key idea promoted by decoherence is the insight that realistic quantum systems are never isolated, but are immersed in the surrounding environment and interact continuously with it. The decoherence program then studies, entirely within the standard quantum formalism (i.e., without adding any new elements in the mathematical theory or its interpretation), the resulting formation of quantum correlations between the states of the system and its environment and the often surprising effects of these system-environment interactions. In short, decoherence brings

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about a local suppression of interference between preferred states selected by the interaction with the environment.

Bub (1997) termed decoherence part of the "new orthodoxy" of understanding quantum mechanics—as the working physicist's way of motivating the postulates of quantum mechanics from physical principles. Proponents of decoherence called it an "historical accident" (Joos, 2000, p. 13) that the implications for quantum mechanics and for the associated foundational problems were overlooked for so long. Zurek (2003b, p. 717) suggests

> The idea that the "openness" of quantum systems might have anything to do with the transition from quantum to classical was ignored for a very long time, probably because in classical physics problems of fundamental importance were always settled in isolated systems.

When the concept of decoherence was first introduced to the broader scientific community by Zurek's (1991) article in *Physics Today*, it elicited a series of contentious comments from the readership (see the April 1993 issue of *Physics Today*). In response to his critics, Zurek (2003b, p. 718) states

> In a field where controversy has reigned for so long this resistance to a new paradigm [namely, to decoherence] is no surprise.

Omnès (2002, p. 2) had this assessment:

> The discovery of decoherence has already much improved our understanding of quantum mechanics. (...) [B]ut its foundation, the range of its validity and its full meaning are still rather obscure. This is due most probably to the fact that it deals with deep aspects of physics, not yet fully investigated.

In particular, the question whether decoherence provides, or at least suggests, a solution to the measurement problem of quantum mechanics has been discussed for several years. For example, Anderson (2001, p. 492) writes in an essay review

> The last chapter (...) deals with the quantum measurement problem (...). My main test, allowing me to bypass the extensive discussion, was a quick, unsuccessful search in the index for the word "decoherence" which describes the process that used to be called "collapse of the wave function."

Zurek speaks in various places of the "apparent" or "effective" collapse of the wave function induced by the interaction with environment (when embedded into a minimal additional interpretive framework) and concludes (Zurek, 1998, p. 1793)

> A "collapse" in the traditional sense is no longer necessary. (...) [The] emergence of "objective existence" [from decoherence] (...) significantly reduces and perhaps even eliminates the role of the "collapse" of the state vector.

D'Espagnat, who considers the explanation of our experiences (i.e., of "appearances") as the only "sure" requirement of a physical theory, states (d'Espagnat, 2000, p. 136)

> For macroscopic systems, the appearances are those of a classical world (no interferences etc.), even in circumstances, such as those occurring in quantum measurements, where quantum effects take place and quantum probabilities intervene (...). Decoherence explains the just mentioned appearances and this is a most important result. (...) As long as we remain within the realm of mere predictions concerning what we shall observe (i.e., what will appear to us)—and refrain from stating anything concerning "things as they must be before we observe them"—no break in the linearity of quantum dynamics is necessary.

In his monumental book on the foundations of quantum mechanics (QM), Auletta (2000, p. 791) concludes that

> the Measurement theory could be part of the interpretation of QM only to the extent that it would still be an open problem, and we think that this is largely no longer the case.

This is mainly so because, according to Auletta (2000, p. 289),

> decoherence is able to solve practically all the problems of Measurement which have been discussed in the previous chapters.

On the other hand, even leading adherents of decoherence have expressed caution or even doubt that decoherence has solved the measurement problem. Joos (2000, p. 14) writes

> Does decoherence solve the measurement problem? Clearly not. What decoherence tells us, is that certain objects appear classical when they are observed. But what is an observation? At some stage, we still have to apply the usual probability rules of quantum theory.

Along these lines, Kiefer and Joos (1999, p. 5) warn that

> One often finds explicit or implicit statements to the effect that the above processes are equivalent to the collapse of the wave function (or even solve the measurement problem). Such statements are certainly unfounded.

In a response to Anderson's (2001, p. 492) comment, Adler (2003, p. 136) states

> I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory.

Similarly, Bacciagaluppi (2003b, p. 3) writes

> Claims that *simultaneously* the measurement problem is real [and] decoherence solves it are confused at best.

Zeh asserts (Joos *et al.*, 2003, Ch. 2)

> Decoherence by itself does not yet solve the measurement problem (...). This argument is nonetheless found wide-spread in the literature. (...) It does seem that the measurement problem can only be resolved if the Schrödinger dynamics (...) is supplemented by a nonunitary collapse (...).

The key achievements of the decoherence program, apart from their implications for conceptual problems, do not seem to be universally understood either. Zurek (1998, p. 1800) remarks

> [The] eventual diagonality of the density matrix (...) is a byproduct (...) but not the essence of decoherence. I emphasize this because diagonality of [the density matrix] in some basis has been occasionally (mis-)interpreted as a key accomplishment of decoherence. This is misleading. Any density matrix is diagonal in some basis. This has little bearing on the interpretation.

These remarks show that a balanced discussion of the key features of decoherence and their implications for the foundations of quantum mechanics is overdue. The decoherence program has made great progress over the past decade, and it would be inappropriate to ignore its relevance in tackling conceptual problems. However, it is equally important to realize the limitations of decoherence in providing consistent and noncircular answers to foundational questions.

An excellent review of the decoherence program has recently been given by Zurek (2003b). It deals primarily with the technicalities of decoherence, although it contains some discussion on how decoherence can be employed in the context of a relative-state interpretation to motivate basic postulates of quantum mechanics. A helpful first orientation and overview, the entry by Bacciagaluppi (2003a) in the *Stanford Encyclopedia of Philosophy* features a relatively short (in comparison to the present paper) introduction to the role of decoherence in the foundations of quantum mechanics, including comments on the relationship between decoherence and several popular interpretations of quantum theory. In spite of these valuable recent contributions to the literature, a detailed and self-contained discussion of the role of decoherence in the foundations of quantum mechanics seems still to be lacking. This review article is intended to fill the gap.

To set the stage, we shall first, in Sec. II, review the measurement problem, which illustrates the key difficulties that are associated with describing quantum measurement within the quantum formalism and that are all in some form addressed by the decoherence program. In Sec. III, we then introduce and discuss the main features of the theory of decoherence, with a particular emphasis on their foundational implications. Finally, in Sec. IV, we investigate the role of decoherence in various interpretive approaches of quantum mechanics, in particular with respect to the ability to motivate and support (or disprove) possible solutions to the measurement problem.

II. THE MEASUREMENT PROBLEM

One of the most revolutionary elements introduced into physical theory by quantum mechanics is the superposition principle, mathematically founded in the linearity of the Hilbert state space. If $|1\rangle$ and $|2\rangle$ are two states, then quantum mechanics tells us that any linear combination $\alpha|1\rangle + \beta|2\rangle$ also corresponds to a possible state. Whereas such superpositions of states have been experimentally extensively verified for microscopic systems (for instance, through the observation of interference effects), the application of the formalism to macroscopic systems appears to lead immediately to severe clashes with our experience of the everyday world. A book has never been ever observed to be in a state of being both "here" and "there" (i.e., to be in a superposition of macroscopically distinguishable positions), nor does a Schrödinger cat that is a superposition of being alive and dead bear much resemblence to reality as we perceive it. The problem is, then, how to reconcile the vastness of the Hilbert space of possible states with the observation of a comparatively few "classical" macrosopic states, defined by having a small number of determinate and robust properties such as position and momentum. Why does the world appear classical to us, in spite of its supposed underlying quantum nature, which would, in principle, allow for arbitrary superpositions?

A. Quantum measurement scheme

This question is usually illustrated in the context of quantum measurement where microscopic superpositions are, via quantum entanglement, amplified into the macroscopic realm and thus lead to very "nonclassical" states that do not seem to correspond to what is actually perceived at the end of the measurement. In the ideal measurement scheme devised by von Neumann (1932), a (typically microscopic) system $\mathcal{S}$, represented by basis vectors $\{|s_n\rangle\}$ in a Hilbert space $\mathcal{H}_{\mathcal{S}}$, interacts with a measurement apparatus $\mathcal{A}$, described by basis vectors $\{|a_n\rangle\}$ spanning a Hilbert space $\mathcal{H}_{\mathcal{A}}$, where the $|a_n\rangle$ are assumed to correspond to macroscopically distinguishable "pointer" positions that correspond to the outcome of a measurement if $\mathcal{S}$ is in the state $|s_n\rangle$.[1]

Now, if $\mathcal{S}$ is in a (microscopically "unproblematic") superposition $\sum_n c_n|s_n\rangle$, and $\mathcal{A}$ is in the initial "ready" state $|a_r\rangle$, the linearity of the Schrödinger equation entails that the total system $\mathcal{SA}$, assumed to be represented by the Hilbert product space $\mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{A}$, evolves according to

$$\left(\sum_n c_n|s_n\rangle\right)|a_r\rangle \quad \stackrel{t}{\longrightarrow} \quad \sum_n c_n|s_n\rangle|a_n\rangle. \tag{2.1}$$

This dynamical evolution is often referred to as a *premeasurement* in order to emphasize that the process described by Eq. (2.1) does not suffice to directly conclude that a measurement has actually been completed. This is so for two reasons. First, the right-hand side is a *superposition* of system-apparatus states. Thus, without supplying an additional physical process (say, some collapse mechanism) or giving a suitable interpretation of such a superposition, it is not clear how to account, given the final composite state, for the definite pointer positions that are perceived as the result of an actual measurement—i.e., why do we seem to perceive the pointer to be in one position $|a_n\rangle$ but not in a superposition of positions? This is the *problem of definite outcomes*. Second, the expansion of the final composite state is in general not unique, and therefore the measured observable is not uniquely defined either. This is the *problem of the preferred basis*. In the literature, the first difficulty is typically referred to as the measurement problem, but the preferred-basis problem is at least equally important, since it does not make sense even to inquire about specific outcomes if the set of possible outcomes is not clearly defined. We shall therefore regard the measurement problem as composed of both the problem of definite outcomes and the problem of the preferred basis, and discuss these components in more detail in the following.

B. The problem of definite outcomes

1. Superpositions and ensembles

The right-hand side of Eq. (2.1) implies that after the premeasurement the combined system $\mathcal{SA}$ is left in a pure state that represents a linear superposition of system-pointer states. It is a well-known and important property of quantum mechanics that a superposition of states is fundamentally different from a classical ensemble of states, where the system actually is in only one of the states but we simply do not know in which (this is often referred to as an "ignorance-interpretable," or "proper" ensemble).

This can be shown explicitely, especially on microscopic scales, by performing experiments that lead to the direct observation of interference patterns instead of the realization of one of the terms in the superposed pure state, for example, in a setup where electrons pass individually (one at a time) through a double slit. As is well known, this experiment clearly shows that, within the standard quantum-mechanical formalism, the electron must not be described by either one of the wave functions describing the passage through a particular slit (ψ_1 *or* ψ_2), but only by the superposition of these wave functions ($\psi_1 + \psi_2$), since the correct density distribution ϱ of the pattern on the screen is not given by the sum of the squared wave functions describing the addition of individual passages through a single slit ($\varrho = |\psi_1|^2 + |\psi_2|^2$), but only by the square of the sum of the individual wave functions ($\varrho = |\psi_1 + \psi_2|^2$).

Put differently, if an ensemble interpretation could be attached to a superposition, the latter would simply represent an ensemble of more fundamentally determined states, and based on the additional knowledge brought about by the results of measurements, we could simply choose a subensemble consisting of the definite pointer state obtained in the measurement. But then, since the time evolution has been strictly deterministic according to the Schrödinger equation, we could backtrack this subensemble in time and thus also specify the initial state more completely ("postselection"), and therefore this state necessarily could not be physically identical to the initially prepared state on the left-hand side of Eq. (2.1).

2. Superpositions and outcome attribution

In the standard ("orthodox") interpretation of quantum mechanics, an observable corresponding to a physical quantity has a definite value if and only if the system is in an eigenstate of the observable; if the system is, however, in a superposition of such eigenstates, as in Eq. (2.1), it is, according to the orthodox interpretation, meaningless to speak of the state of the system as having any definite value of the observable at all. (This is frequently referred to as the so-called eigenvalue-eigenstate link, or "e-e link" for short.) The e-e link, however, is by no means forced upon us by the structure of quantum mechanics or by empirical constraints (Bub, 1997). The concept of (classical) "values" that can be ascribed through the e-e link based on observables and the existence of exact eigenstates of these observables has therefore frequently been either weakened or altogether abandonded. For instance, outcomes of measurements are typically registered in position space (pointer positions, etc.), but there exist no exact eigenstates of the position operator, and the pointer states are never exactly mutually orthogonal. One might then (explicitely or implic-

[1] Note that von Neumann's scheme is in sharp contrast to the Copenhagen interpretation, where measurement is not treated as a system-apparatus interaction described by the usual quantum-mechanical formalism, but instead as an independent component of the theory, to be represented entirely in fundamentally classical terms.

itly) promote a "fuzzy" e-e link, or give up the concept of observables and values entirely and directly interpret the time-evolved wave functions (working in the Schrödinger picture) and the corresponding density matrices. Also, if it is regarded as sufficient to explain our perceptions rather than describe the "absolute" state of the entire universe (see the argument below), one might only require that the (exact or fuzzy) e-e link hold in a "relative" sense, i.e., for the state of the rest of the universe relative to the state of the observer.

Then, to solve the problem of definite outcomes, some interpretations (for example, modal interpretations and relative-state interpretations) interpret the final-state superposition in such a way as to explain the existence, or at least the subjective perception, of "outcomes" even if the final composite state has the form of a superposition. Other interpretations attempt to solve the measurement problem by modifying the strictly unitary Schrödinger dynamics. Most prominently, the orthodox interpretation postulates a collapse mechanism that transforms a pure-state density matrix into an ignorance-interpretable ensemble of individual states (a "proper mixture"). Wave-function collapse theories add stochastic terms to the Schrödinger equation that induce an effective (albeit only approximate) collapse for states of macroscopic systems (Ghirardi *et al.*, 1986; Gisin, 1984; Pearle, 1979, 1999), while other authors suggested that collapse occurs at the level of the mind of a conscious observer (Stapp, 1993; Wigner, 1963). Bohmian mechanics, on the other hand, upholds a unitary time evolution of the wavefunction, but introduces an additional dynamical law that explicitely governs the always-determinate positions of all particles in the system.

3. Objective vs. subjective definiteness

In general, (macroscopic) definiteness—and thus a solution to the problem of outcomes in the theory of quantum measurement—can be achieved either on an *ontological* (objective) or an *observational* (subjective) level. Objective definiteness aims at ensuring "actual" definiteness in the macroscopic realm, whereas subjective definiteness only attempts to explain why the macroscopic world appears to be definite—and thus does not make any claims about definiteness of the underlying physical reality (whatever this reality might be). This raises the question of the significance of this distinction with respect to the formation of a satisfactory theory of the physical world. It might appear that a solution to the measurement problem based on ensuring subjective, but not objective, definiteness is merely good "for all practical purposes"—abbreviated, rather disparagingly, as "FAPP" by Bell (1990)—and thus not capable of solving the "fundamental" problem that would seem relevant to the construction of the "precise theory" that Bell demanded so vehemently.

It seems to the author, however, that this critism is not justified, and that subjective definiteness should be viewed on a par with objective definitess with respect to a satisfactory solution to the measurement problem. We demand objective definiteness because we experience definiteness on the subjective level of observation, and it should not be viewed as an *a priori* requirement for a physical theory. If we knew independently of our experience that definiteness existed in nature, subjective definiteness would presumably follow as soon as we had employed a simple model that connected the "external" physical phenomena with our "internal" perceptual and cognitive apparatus, where the expected simplicity of such a model can be justified by referring to the presumed identity of the physical laws governing external and internal processes. But since knowledge is based on experience, that is, on observation, the existence of objective definiteness could only be derived from the observation of definiteness. And, moreover, observation tells us that definiteness is in fact not a universal property of nature, but rather a property of macroscopic objects, where the borderline to the macroscopic realm is difficult to draw precisely; mesoscopic interference experiments have demonstrated clearly the blurriness of the boundary. Given the lack of a precise definition of the boundary, any demand for fundamental definiteness on the objective level should be based on a much deeper and more general commitment to a definiteness that applies to every physical entity (or system) across the board, regardless of spatial size, physical property, and the like.

Therefore, if we realize that the often deeply felt commitment to a general objective definiteness is only based on our experience of macroscopic systems, and that this definiteness in fact fails in an observable manner for microscopic and even certain mesoscopic systems, the author sees no compelling grounds on which objective definiteness must be demanded as part of a satisfactory physical theory, provided that the theory can account for subjective, observational definiteness in agreement with our experience. Thus the author suggests that the same legitimacy be attributed to proposals for a solution of the measurement problem that achieve "only" subjective but not objective definiteness—after all, the measurement problem arises solely from a clash of our experience with certain implications of the quantum formalism. D'Espagnat (2000, pp. 134–135) has advocated a similar viewpoint:

> The fact that we perceive such "things" as macroscopic objects lying at distinct places is due, partly at least, to the structure of our sensory and intellectual equipment. We should not, therefore, take it as being part of the body of sure knowledge that we have to take into account for defining a quantum state. (. . .) In fact, scientists most righly claim that the purpose of science is to describe human experience, not to describe "what really is"; and as long as we only want to describe human experience, that is, as long as we are content with being able to predict what will

> be observed in all possible circumstances (...) we need not postulate the existence—in some absolute sense—of unobserved (i.e., not yet observed) objects lying at definite places in ordinary 3-dimensional space.

C. The preferred-basis problem

The second difficulty associated with quantum measurement is known as the preferred-basis problem, which demonstrates that the measured observable is in general not uniquely defined by Eq. (2.1). For any choice of system states $\{|s_n\rangle\}$, we can find corresponding apparatus states $\{|a_n\rangle\}$, and vice versa, to equivalently rewrite the final state emerging from the premeasurement interaction, i.e., the right-hand side of Eq. (2.1). In general, however, for some choice of apparatus states the corresponding new system states will not be mutually orthogonal, so that the observable associated with these states will not be Hermitian, which is usually not desired (however, not forbidden—see the discussion by Zurek, 2003b). Conversely, to ensure distinguishable outcomes, we must, in general, require the (at least approximate) orthogonality of the apparatus (pointer) states, and it then follows from the biorthogonal decomposition theorem that the expansion of the final premeasurement system-apparatus state of Eq. (2.1),

$$|\psi\rangle = \sum_n c_n |s_n\rangle |a_n\rangle, \tag{2.2}$$

is unique, but only if all coefficients c_n are distinct. Otherwise, we can in general rewrite the state in terms of different state vectors,

$$|\psi\rangle = \sum_n c'_n |s'_n\rangle |a'_n\rangle, \tag{2.3}$$

such that the same postmeasurement state seems to correspond to two different measurements, that is, of the observables $\widehat{A} = \sum_n \lambda_n |s_n\rangle\langle s_n|$ and $\widehat{B} = \sum_n \lambda'_n |s'_n\rangle\langle s'_n|$ of the system, respectively, although in general $\widehat{A}$ and $\widehat{B}$ do not commute.

As an example, consider a Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ where $\mathcal{H}_1$ and $\mathcal{H}_2$ are two-dimensional spin spaces with states corresponding to spin up or spin down along a given axis. Suppose we are given an entangled spin state of the Einstein-Podolsky-Rosen form (Einstein *et al.*, 1935)

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|z+\rangle_1 |z-\rangle_2 - |z-\rangle_1 |z+\rangle_2), \tag{2.4}$$

where $|z\pm\rangle_{1,2}$ represents the eigenstates of the observable σ_z corresponding to spin up or spin down along the z axis of the two systems 1 and 2. The state $|\psi\rangle$ can however equivalently be expressed in the spin basis corresponding to any other orientation in space. For example, when using the eigenstates $|x\pm\rangle_{1,2}$ of the observable σ_x (which represents a measurement of the spin orientation along the x axis) as basis vectors, we get

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x+\rangle_1 |x-\rangle_2 - |x-\rangle_1 |x+\rangle_2). \tag{2.5}$$

Now suppose that system 2 acts as a measuring device for the spin of system 1. Then Eqs. (2.4) and (2.5) imply that the measuring device has established a correlation with both the z and the x spin of system 1. This means that, if we interpret the formation of such a correlation as a measurement in the spirit of the von Neumann scheme (without assuming a collapse), our apparatus (system 2) could be considered as having measured also the x spin once it has measured the z spin, and vice versa—in spite of the noncommutativity of the corresponding spin observables σ_z and σ_x. Moreover, since we can rewrite Eq. (2.4) in infinitely many ways, it appears that once the apparatus has measured the spin of system 1 along one direction, it can also be regarded as having measured the spin along any other direction, again in apparent contradiction with quantum mechanics due to the noncommutativity of the spin observables corresponding to different spatial orientations.

It thus seems that quantum mechanics has nothing to say about which observable(s) of the system is (are) being recorded, via the formation of quantum correlations, by the apparatus. This can be stated in a general theorem (Auletta, 2000; Zurek, 1981): When quantum mechanics is applied to an isolated composite object consisting of a system $\mathcal{S}$ and an apparatus $\mathcal{A}$, it cannot determine which observable of the system has been measured—in obvious contrast to our experience of the workings of measuring devices that seem to be "designed" to measure certain quantities.

D. The quantum-to-classical transition and decoherence

In essence, as we have seen above, the measurement problem deals with the transition from a quantum world, described by essentially arbitrary linear superpositions of state vectors, to our perception of "classical" states in the macroscopic world, that is, a comparatively small subset of the states allowed by the quantum-mechanical superposition principle, having only a few, but determinate and robust, properties, such as position, momentum, etc. The question of why and how our experience of a "classical" world emerges from quantum mechanics thus lies at the heart of the foundational problems of quantum theory.

Decoherence has been claimed to provide an explanation for this *quantum-to-classical transition* by appealing to the ubiquitous immersion of virtually all physical systems in their environment ("environmental monitoring"). This trend can also be read off nicely from the titles of some papers and books on decoherence, for example, "The emergence of classical properties through interaction with the environment" (Joos and Zeh, 1985), "Decoherence and the transition from quantum to classical" (Zurek, 1991), and "Decoherence and the appearance of a classical world in quantum theory" (Joos *et al.*, 2003). We shall critically investigate in this paper to what extent the appeal to decoherence for an explanation of the quantum-to-classical transition is justified.

III. THE DECOHERENCE PROGRAM

As remarked earlier, the theory of decoherence is based on a study of the effects brought about by the interaction of physical systems with their environment. In classical physics, the environment is usually viewed as a kind of disturbance, or noise, that perturbs the system under consideration in such a way as to negatively influence the study of its "objective" properties. Therefore science has established the idealization of isolated systems, with experimental physics aiming at eliminating any outer sources of disturbance as much as possible in order to discover the "true" underlying nature of the system under study.

The distinctly nonclassical phenomenon of quantum entanglement, however, has demonstrated that the correlations between two systems can be of fundamental importance and can lead to properties that are not present in the individual systems.[2] The earlier view of phenomena arising from quantum entanglement as "paradoxa" has generally been replaced by the recognition of entanglement as a fundamental property of nature.

The decoherence program[3] is based on the idea that such quantum correlations are ubiquitous; that nearly every physical system must interact in some way with its environment (for example, with the surrounding photons that then create the visual experience within the observer), which typically consists of a large number of degrees of freedom that are hardly ever fully controlled. Only in very special cases of typically microscopic (atomic) phenomena, so goes the claim of the decoherence program, is the idealization of isolated systems applicable so that the predictions of linear quantum mechanics (i.e., a large class of superpositions of states) can actually be observationally confirmed. In the majority of the cases accessible to our experience, however, interaction with the environment is so dominant as to preclude the observation of the "pure" quantum world, imposing effective superselection rules (Cisnerosy *et al.*, 1998; Galindo *et al.*, 1962; Giulini, 2000; Wick *et al.*, 1952, 1970; Wightman, 1995) onto the space of observable states that lead to states corresponding to the "classical" properties of our experience. Interference between such states gets locally suppressed and is thus claimed to become inaccessible to the observer.

Probably the most surprising aspect of decoherence is the effectiveness of the system-environment interactions. Decoherence typically takes place on extremely short time scales and requires the presence of only a minimal environment (Joos and Zeh, 1985). Due to the large number of degrees of freedom of the environment, it is usually very difficult to undo system-environment entanglement, which has been claimed as a source of our impression of irreversibility in nature (see, for example, Kiefer and Joos, 1999; Zeh, 2001; Zurek, 1982, 2003b; Zurek and Paz, 1994). In general, the effect of decoherence increases with the size of the system (from microscopic to macroscopic scales), but it is important to note that there exist, admittedly somewhat exotic, examples for which the decohering influence of the environment can be sufficiently shielded to lead to mesoscopic and even macroscopic superpositions. One such example would be the case of superconducting quantum interference devices (SQUIDs), in which superpositions of macroscopic currents become observable (Friedman *et al.*, 2000; van der Wal *et al.*, 2000). Conversely, some microscopic systems (for instance, certain chiral molecules that exist in different distinct spatial configurations) can be subject to remarkably strong decoherence.

The decoherence program has dealt with the following two main consequences of environmental interaction:

(1) *Environment-induced decoherence.* The fast local suppression of interference between different states of the system. However, since only unitary time evolution is employed, global phase coherence is not actually destroyed—it becomes absent from the local density matrix that describes the system alone, but remains fully present in the total system-environment composition.[4] We shall discuss environment-induced local decoherence in more detail in Sec. III.D.

(2) *Environment-induced superselection.* The selection of preferred sets of states, often referred to as "pointer states," that are robust (in the sense of retaining correlations over time) in spite of their immersion in the environment. These states are determined by the form of the interaction between the system and its environment and are suggested to correspond to the "classical" states of our experience. We shall consider this mechanism in Sec. III.E.

[2] Broadly speaking, this means that the (quantum-mechanical) whole is different from the sum of its parts.

[3] For key ideas and concepts, see Joos and Zeh (1985); Joos *et al.* (2003); Kübler and Zeh (1973); Zeh (1970, 1973, 1995, 1997, 2000); Zurek (1981, 1982, 1991, 1993, 2003b).

[4] Note that the persistence of coherence in the total state is important to ensure the possibility of describing special cases in which mesoscopic or macrosopic superpositions have been experimentally realized.

Another, more recent aspect of the decoherence program, termed *enviroment-assisted invariance* or "envariance," was introduced by Zurek (2003b,c, 2004b) and further developed in Zurek (2004a). In particular, Zurek used envariance to explain the emergence of probabilities in quantum mechanics and to derive Born's rule based on certain assumptions. We shall review envariance and Zurek's derivation of the Born rule in Sec. III.F.

Finally, let us emphasize that decoherence arises from a direct application of the quantum mechanical formalism to a description of the interaction of a physical system with its environment. By itself, decoherence is therefore neither an interpretation nor a modification of quantum mechanics. Yet the implications of decoherence need to be interpreted in the context of the different interpretations of quantum mechanics. Also, since decoherence effects have been studied extensively in both theoretical models and experiments (for a survey, see, for example, Joos *et al.*, 2003; Zurek, 2003b), their existence can be taken as a well-confirmed fact.

A. Resolution into subsystems

Note that decoherence derives from the presupposition of the existence and the possibility of a division of the world into "system(s)" and "environment." In the decoherence program, the term "environment" is usually understood as the "remainder" of the system, in the sense that its degrees of freedom are typically not (cannot be, do not need to be) controlled and are not directly relevant to the observation under consideration (for example, the many microsopic degrees of freedom of the system), but that nonetheless the environment includes "all those degrees of freedom which contribute significantly to the evolution of the state of the apparatus" (Zurek, 1981, p. 1520).

This system–environment dualism is generally associated with quantum entanglement, which always describes a correlation between parts of the universe. As long as the universe is not resolved into individual subsystems, there is no measurement problem: the state vector $|\Psi\rangle$ of the entire universe[5] evolves deterministically according to the Schrödinger equation $i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \widehat{H}|\Psi\rangle$, which poses no interpretive difficulty. Only when we decompose the total Hilbert-state space $\mathcal{H}$ of the universe into a product of two spaces $\mathcal{H}_1 \otimes \mathcal{H}_2$, and accordingly form the joint-state vector $|\Psi\rangle = |\Psi_1\rangle|\Psi_2\rangle$, and want to ascribe an individual state (besides the joint state that describes a correlation) to one of the two systems (say, the apparatus), does the measurement problem arise. Zurek (2003b, p. 718) puts it like this:

> In the absence of systems, the problem of interpretation seems to disappear. There is simply no need for "collapse" in a universe with no systems. Our experience of the classical reality does not apply to the universe as a whole, seen from the outside, but to the systems within it.

Moreover, terms like "observation," "correlation," and "interaction" will naturally make little sense without a division into systems. Zeh has suggested that the locality of the observer defines an observation in the sense that any observation arises from the ignorance of a part of the universe; and that this also defines the "facts" that can occur in a quantum system. Landsman (1995, pp. 45–46) argues similarly:

> The essence of a "measurement," "fact" or "event" in quantum mechanics lies in the non-observation, or irrelevance, of a certain part of the system in question. (. . .) A world without parts declared or forced to be irrelevant is a world without facts.

However, the assumption of a decomposition of the universe into subsystems—as necessary as it appears to be for the emergence of the measurement problem and for the definition of the decoherence program—is definitely nontrivial. By definition, the universe as a whole is a closed system, and therefore there are no "unobserved degrees of freedom" of an external environment which would allow for the application of the theory of decoherence to determine the space of quasiclassical observables of the universe in its entirety. Also, there exists no general criterion for how the total Hilbert space is to be divided into subsystems, while at the same time much of what is called a property of the system will depend on its correlation with other systems. This problem becomes particularly acute if one would like decoherence not only to motivate explanations for the subjective perception of classicality (as in Zurek's "existential interpretation," see Zurek, 1993, 1998, 2003b, and Sec. IV.C below), but moreover to allow for the definition of quasiclassical "macrofacts." Zurek (1998, p. 1820) admits this severe conceptual difficulty:

> In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the "systems" which play such a crucial role in all the discussions of the emergent classicality. (. . .) [A] compelling explanation of what are the systems—how to define them given, say, the overall Hamiltonian in some suitably large Hilbert space—would be undoubtedly most useful.

A frequently proposed idea is to abandon the notion of an "absolute" resolution and instead postulate the intrinsic relativity of the distinct state spaces and properties that emerge through the correlation between these relatively

[5] If we dare to postulate this total state—see counterarguments by Auletta (2000).

defined spaces (see, for example, the proposals, unrelated to decoherence, of Everett, 1957; Mermin, 1998a,b; and Rovelli, 1996). This relative view of systems and correlations has counterintuitive, in the sense of nonclassical, implications. However, as in the case of quantum entanglement, these implications need not be taken as paradoxa that demand further resolution. Accepting some properties of nature as counterintuitive is indeed a satisfactory path to take in order to arrive at a description of nature that is as complete and objective as is allowed by the range of our experience (which is based on inherently local observations).

B. The concept of reduced density matrices

Since reduced density matrices are a key tool of decoherence, it will be worthwile to briefly review their basic properties and interpretation in the following. The concept of reduced density matrices emerged in the earliest days of quantum mechanics (Furry, 1936; Landau, 1927; von Neumann, 1932; for some historical remarks, see Pessoa, 1998). In the context of a system of two entangled systems in a pure state of the Einstein-Podolsky-Rosen-type,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2), \tag{3.1}$$

it had been realized early that for an observable $\widehat{O}$ that pertains only to system 1, $\widehat{O} = \widehat{O}_1 \otimes \widehat{I}_2$, the pure-state density matrix $\rho = |\psi\rangle\langle\psi|$ yields, according to the trace rule $\langle\widehat{O}\rangle = \mathrm{Tr}(\rho\widehat{O})$ and given the usual Born rule for calculating probabilities, exactly the same statistics as the reduced density matrix ρ_1 obtained by tracing over the degrees of freedom of system 2 (i.e., the states $|+\rangle_2$ and $|-\rangle_2$),

$$\rho_1 = \mathrm{Tr}_2|\psi\rangle\langle\psi| = {}_2\langle+|\psi\rangle\langle\psi|+\rangle_2 + {}_2\langle-|\psi\rangle\langle\psi|-\rangle_2, \tag{3.2}$$

since it is easy to show that, for this observable $\widehat{O}$,

$$\langle\widehat{O}\rangle_\psi = \mathrm{Tr}(\rho\widehat{O}) = \mathrm{Tr}_1(\rho_1\widehat{O}_1). \tag{3.3}$$

This result holds in general for any pure state $|\psi\rangle = \sum_i \alpha_i|\phi_i\rangle_1|\phi_i\rangle_2\cdots|\phi_i\rangle_N$ of a resolution of a system into N subsystems, where the $\{|\phi_i\rangle_j\}$ are assumed to form orthonormal basis sets in their respective Hilbert spaces $\mathcal{H}_j$, $j = 1\cdots N$. For any observable $\widehat{O}$ that pertains only to system j, $\widehat{O} = \widehat{I}_1\otimes\widehat{I}_2\otimes\cdots\otimes\widehat{I}_{j-1}\otimes\widehat{O}_j\otimes\widehat{I}_{j+1}\otimes\cdots\otimes\widehat{I}_N$, the statistics of $\widehat{O}$ generated by applying the trace rule will be identical regardless of whether we use the pure-state density matrix $\rho = |\psi\rangle\langle\psi|$ or the reduced density matrix $\rho_j = \mathrm{Tr}_{1,\ldots,j-1,j+1,\ldots,N}|\psi\rangle\langle\psi|$, since again $\langle\widehat{O}\rangle = \mathrm{Tr}(\rho\widehat{O}) = \mathrm{Tr}_j(\rho_j\widehat{O}_j)$.

The typical situation in which the reduced density matrix arises is this: Before a premeasurement-type interaction, the observer knows that each individual system is in some (unknown) pure state. After the interaction, i.e., after the correlation between the systems is established, the observer has access to only one of the systems, say, system 1; everything that can be known about the state of the composite system must therefore be derived from measurements on system 1, which will yield the possible outcomes of system 1 and their probability distribution. All information that can be extracted by the observer is then, exhaustively and correctly, contained in the reduced density matrix of system 1, assuming that the Born rule for quantum probabilities holds.

Let us return to the Einstein-Podolsky-Rosen-type example, Eqs. (3.1) and (3.2). If we assume that the states of system 2 are orthogonal, ${}_2\langle+|-\rangle_2 = 0$, ρ_1 becomes diagonal,

$$\rho_1 = \mathrm{Tr}_2|\psi\rangle\langle\psi| = \frac{1}{2}(|+\rangle\langle+|)_1 + \frac{1}{2}(|-\rangle\langle-|)_1. \tag{3.4}$$

But this density matrix is formally identical to the density matrix that would be obtained if system 1 were in a mixed state, i.e., in either one of the two states $|+\rangle_1$ and $|-\rangle_1$ with equal probabilties—as opposed to the superposition $|\psi\rangle$, in which both terms are considered present, which could in principle be confirmed by suitable interference experiments. This implies that a measurement of an observable that only pertains to system 1 cannot discriminate between the two cases, pure vs mixed state.[6]

However, note that the formal identification of the reduced density matrix with a mixed-state density matrix is easily misinterpreted as implying that the state of the system can be viewed as mixed too (see also the discussion by d'Espagnat, 1988). Density matrices are only a calculational tool for computing the probability distribution of a set of possible outcomes of measurements; they do not specify the state of the system.[7] Since the two systems are entangled and the total composite system is still described by a superposition, it follows from the standard rules of quantum mechanics that no individual definite state can be attributed to one of the systems. The reduced density matrix looks like a mixed-state density matrix because, if one actually measured an observable of the system, one would expect to get a definite outcome with a certain probability; in terms of measurement statistics, this is equivalent to the situation in which the system is in one of the states from the set of possible outcomes from the beginning, that is, before the measurement. As Pessoa (1998, p. 432) puts it, "taking a partial trace amounts to the statistical version of the projection postulate."

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[6] As discussed by Bub (1997, pp. 208–210), this result also holds for any observable of the composite system that factorizes into the form $\widehat{O} = \widehat{O}_1 \otimes \widehat{O}_2$, where $\widehat{O}_1$ and $\widehat{O}_2$ do not commute with the projection operators $(|\pm\rangle\langle\pm|)_1$ and $(|\pm\rangle\langle\pm|)_2$, respectively.

[7] In this context we note that any nonpure density matrix can be written in many different ways, demonstrating that any partition in a particular ensemble of quantum states is arbitrary.

C. A modified von Neumann measurement scheme

Let us now reconsider the von Neumann model for ideal quantum-mechanical measurement, Eq. (3.5), but now with the environment included. We shall denote the environment by $\mathcal{E}$ and represent its state before the measurement interaction by the initial state vector $|e_0\rangle$ in a Hilbert space $\mathcal{H}_\mathcal{E}$. As usual, let us assume that the state space of the composite object system-apparatus-environment is given by the tensor product of the individual Hilbert spaces, $\mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{E}$. The linearity of the Schrödinger equation then yields the following time evolution of the entire system $\mathcal{SAE}$,

$$\left(\sum_n c_n|s_n\rangle\right)|a_r\rangle|e_0\rangle \xrightarrow{(1)} \left(\sum_n c_n|s_n\rangle|a_n\rangle\right)|e_0\rangle \xrightarrow{(2)} \sum_n c_n|s_n\rangle|a_n\rangle|e_n\rangle, \quad (3.5)$$

where the $|e_n\rangle$ are the states of the environment associated with the different pointer states $|a_n\rangle$ of the measuring apparatus. Note that while for two subsystems, say, $\mathcal{S}$ and $\mathcal{A}$, there always exists a diagonal ("Schmidt") decomposition of the final state of the form $\sum_n c_n|s_n\rangle|a_n\rangle$, for three subsystems (for example, $\mathcal{S}$, $\mathcal{A}$, and $\mathcal{E}$), a decomposition of the form $\sum_n c_n|s_n\rangle|a_n\rangle|e_n\rangle$ is not always possible. This implies that the total Hamiltonian that induces a time evolution of the above kind, Eq. (3.5), must be of a special form.[8]

Typically, the $|e_n\rangle$ will be product states of many microsopic subsystem states $|\varepsilon_n\rangle_i$ corresponding to the individual parts that form the environment, i.e., $|e_n\rangle = |\varepsilon_n\rangle_1|\varepsilon_n\rangle_2|\varepsilon_n\rangle_3\cdots$. We see that a nonseparable and in most cases, for all practical purposes, irreversible (due to the enormous number of degrees of freedom of the environment) correlation has been established between the states of the system–apparatus combination $\mathcal{SA}$ and the different states of the environment $\mathcal{E}$. Note that Eq. (3.5) also implies that the environment has recorded the state of the system—and, equivalently, the state of the system-apparatus composition. The environment, composed of many subsystems, thus acts as an amplifying, higher-order measuring device.

D. Decoherence and local suppression of interference

Interaction with the environment typically leads to a rapid vanishing of the diagonal terms in the local density matrix describing the probability distribution for the outcomes of measurements on the system. This effect has become known as environment-induced decoherence, and it has also frequently been claimed to imply at least a partial solution to the measurement problem.

1. General formalism

In Sec. III.B, we have already introduced the concept of local (or reduced) density matrices and pointed out some caveats on their interpretation. In the context of the decoherence program, reduced density matrices arise as follows. Any observation will typically be restricted to the system-apparatus component, $\mathcal{SA}$, while the many degrees of freedom of the environment $\mathcal{E}$ remain unobserved. Of course, typically some degrees of freedom of the environment will always be included in our observation (e.g., some of the photons scattered off the apparatus) and we shall accordingly include them in the "observed part $\mathcal{SA}$ of the universe." The crucial point is that there still remains a comparatively large number of environmental degrees of freedom that will not be observed directly.

Suppose then that the operator $\widehat{O}_{\mathcal{SA}}$ represents an observable of $\mathcal{SA}$ only. Its expectation value $\langle\widehat{O}_{\mathcal{SA}}\rangle$ is given by

$$\langle\widehat{O}_{\mathcal{SA}}\rangle = \mathrm{Tr}(\widehat{\rho}_{\mathcal{SAE}}[\widehat{O}_{\mathcal{SA}} \otimes \widehat{I}_\mathcal{E}]) = \mathrm{Tr}_{\mathcal{SA}}(\widehat{\rho}_{\mathcal{SA}}\widehat{O}_{\mathcal{SA}}), \quad (3.6)$$

where the density matrix $\widehat{\rho}_{\mathcal{SAE}}$ of the total $\mathcal{SAE}$ combination,

$$\widehat{\rho}_{\mathcal{SAE}} = \sum_{mn} c_m c_n^*|s_m\rangle|a_m\rangle|e_m\rangle\langle s_n|\langle a_n|\langle e_n|, \quad (3.7)$$

has, for all practical purposes of statistical prediction, been replaced by the local (or reduced) density matrix $\widehat{\rho}_{\mathcal{SA}}$, obtained by "tracing out the unobserved degrees of the environment," that is,

$$\widehat{\rho}_{\mathcal{SA}} = \mathrm{Tr}_\mathcal{E}(\widehat{\rho}_{\mathcal{SAE}}) = \sum_{mn} c_m c_n^*|s_m\rangle|a_m\rangle\langle s_n|\langle a_n|\langle e_n|e_m\rangle. \quad (3.8)$$

So far, $\widehat{\rho}_{\mathcal{SA}}$ contains characteristic interference terms $|s_m\rangle|a_m\rangle\langle s_n|\langle a_n|$, $m \neq n$, since we cannot assume from the outset that the basis vectors $|e_m\rangle$ of the environment are necessarily mutually orthogonal, i.e., that $\langle e_n|e_m\rangle = 0$ if $m \neq n$. Many explicit physical models for the interaction of a system with the environment (see Sec. III.D.2 below for a simple example), however, have shown that due to the large number of subsystems that compose the environment, the pointer states $|e_n\rangle$ of the environment rapidly approach orthogonality, $\langle e_n|e_m\rangle(t) \to \delta_{n,m}$, such that the reduced density matrix $\widehat{\rho}_{\mathcal{SA}}$ becomes approximately orthogonal in the "pointer basis" $\{|a_n\rangle\}$; that is,

$$\widehat{\rho}_{\mathcal{SA}} \xrightarrow{t} \widehat{\rho}_{\mathcal{SA}}^{\,d} \approx \sum_n |c_n|^2|s_n\rangle|a_n\rangle\langle s_n|\langle a_n| = \sum_n |c_n|^2 \widehat{P}_n^{(\mathcal{S})} \otimes \widehat{P}_n^{(\mathcal{A})}. \quad (3.9)$$

[8] For an example of such a Hamiltonian, see the model of Zurek (1981, 1982) and its outline in Sec. III.D.2 below. For a critical comment regarding limitations on the form of the evolution operator and the possibility of a resulting disagreement with experimental evidence, see Pessoa (1998).

Here, $\widehat{P}_n^{(\mathcal{S})}$ and $\widehat{P}_n^{(\mathcal{A})}$ are the projection operators onto the eigenstates of $\mathcal{S}$ and $\mathcal{A}$, respectively. Therefore the interference terms have vanished in this local representation, i.e., phase coherence has been locally lost. This is precisely the effect referred to as environment-induced decoherence. The decohered local density matrices describing the probability distribution of the outcomes of a measurement on the system-apparatus combination is formally (approximately) identical to the corresponding mixed-state density matrix. But as we pointed out in Sec. III.B, we must be careful in interpreting this state of affairs, since full coherence is retained in the total density matrix $\rho_{\mathcal{SAE}}$.

2. An exactly solvable two-state model for decoherence

To see how the approximate mutual orthogonality of the environmental state vectors arises, let us discuss a simple model first introduced by Zurek (1982). Consider a system $\mathcal{S}$ with two spin states $\{|\Uparrow\rangle, |\Downarrow\rangle\}$ that interacts with an environment $\mathcal{E}$ described by a collection of N other two-state spins represented by $\{|\uparrow_k\rangle, |\downarrow_k\rangle\}$, $k = 1 \cdots N$. The self-Hamiltonians $\widehat{H}_{\mathcal{S}}$ and $\widehat{H}_{\mathcal{E}}$ and the self-interaction Hamiltonian $\widehat{H}_{\mathcal{EE}}$ of the environment are taken to be equal to zero. Only the interaction Hamiltonian $\widehat{H}_{\mathcal{SE}}$ that describes the coupling of the spin of the system to the spins of the environment is assumed to be nonzero and of the form

$$\widehat{H}_{\mathcal{SE}} = (|\Uparrow\rangle\langle\Uparrow| - |\Downarrow\rangle\langle\Downarrow|) \otimes \sum_k g_k (|\uparrow_k\rangle\langle\uparrow_k| - |\downarrow_k\rangle\langle\downarrow_k|) \bigotimes_{k' \neq k} \widehat{I}_{k'}, \tag{3.10}$$

where the g_k are coupling constants and $\widehat{I}_k = (|\uparrow_k\rangle\langle\uparrow_k| + |\downarrow_k\rangle\langle\downarrow_k|)$ is the identity operator for the kth environmental spin. Applied to the initial state before the interaction is turned on,

$$|\psi(0)\rangle = (a|\Uparrow\rangle + b|\Downarrow\rangle) \bigotimes_{k=1}^{N} (\alpha_k|\uparrow_k\rangle + \beta_k|\downarrow_k\rangle), \tag{3.11}$$

this Hamiltonian yields a time evolution of the state given by

$$|\psi(t)\rangle = a|\Uparrow\rangle|E_\Uparrow(t)\rangle + b|\Downarrow\rangle|E_\Downarrow(t)\rangle, \tag{3.12}$$

where the two environmental states $|E_\Uparrow(t)\rangle$ and $|E_\Downarrow(t)\rangle$ are

$$|E_\Uparrow(t)\rangle = |E_\Downarrow(-t)\rangle = \bigotimes_{k=1}^{N} (\alpha_k e^{ig_k t}|\uparrow_k\rangle + \beta_k e^{-ig_k t}|\downarrow_k\rangle). \tag{3.13}$$

The reduced density matrix $\rho_{\mathcal{S}}(t) = \mathrm{Tr}_{\mathcal{E}}(|\psi(t)\rangle\langle\psi(t)|)$ is then

$$\begin{aligned} \rho_{\mathcal{S}}(t) &= |a|^2|\Uparrow\rangle\langle\Uparrow| + |b|^2|\Downarrow\rangle\langle\Downarrow| \\ &\quad + z(t)ab^*|\Uparrow\rangle\langle\Downarrow| + z^*(t)a^*b|\Downarrow\rangle\langle\Uparrow|, \end{aligned} \tag{3.14}$$

where the interference coefficient $z(t)$ which determines the weight of the off-diagonal elements in the reduced density matrix is given by

$$z(t) = \langle E_\Uparrow(t)|E_\Downarrow(t)\rangle = \prod_{k=1}^{N} (|\alpha_k|^2 e^{ig_k t} + |\beta_k|^2 e^{-ig_k t}), \tag{3.15}$$

and thus

$$|z(t)|^2 = \prod_{k=1}^{N} \{1 + [(|\alpha_k|^2 - |\beta_k|^2)^2 - 1] \sin^2 2g_k t\}. \tag{3.16}$$

At $t = 0$, $z(t) = 1$, i.e., the interference terms are fully present, as expected. If $|\alpha_k|^2 = 0$ or 1 for each k, i.e., if the environment is in an eigenstate of the interaction Hamiltonian $\widehat{H}_{\mathcal{SE}}$ of the type $|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle \cdots |\uparrow_N\rangle$, and/or if $2g_k t = m\pi$ ($m = 0, 1, \ldots$), then $z(t)^2 \equiv 1$ so coherence is retained over time. However, under realistic circumstances, we can typically assume a random distribution of the initial states of the environment (i.e., of coefficients α_k, β_k) and of the coupling coefficients g_k. Then, in the long-time average,

$$\langle |z(t)|^2 \rangle_{t\to\infty} \simeq 2^{-N} \prod_{k=1}^{N} [1 + (|\alpha_k|^2 - |\beta_k|^2)^2] \stackrel{N\to\infty}{\longrightarrow} 0, \tag{3.17}$$

so the off-diagonal terms in the reduced density matrix become strongly damped for large N.

It can also be shown directly that, given very general assumptions about the distribution of the couplings g_k (namely, requiring their initial distribution to have finite variance), $z(t)$ exhibits a Gaussian time dependence of the form $z(t) \propto e^{iAt}e^{-B^2t^2/2}$, where A and B are real constants (Zurek *et al.*, 2003). For the special case in which $\alpha_k = \alpha$ and $g_k = g$ for all k, this behavior of $z(t)$ can be immediately seen by first rewriting $z(t)$ as the binomial expansion

$$\begin{aligned} z(t) &= (|\alpha|^2 e^{igt} + |\beta|^2 e^{-igt})^N \\ &= \sum_{l=0}^{N} \binom{N}{l} |\alpha|^{2l}|\beta|^{2(N-l)} e^{ig(2l-N)t}. \end{aligned} \tag{3.18}$$

For large N, the binomial distribution can then be approximated by a Gaussian,

$$\binom{N}{l} |\alpha|^{2l}|\beta|^{2(N-l)} \approx \frac{e^{-(l-N|\alpha|^2)^2/(2N|\alpha|^2|\beta|^2)}}{\sqrt{2\pi N|\alpha|^2|\beta|^2}}, \tag{3.19}$$

in which case $z(t)$ becomes

$$z(t) = \sum_{l=0}^{N} \frac{e^{-(l-N|\alpha|^2)^2/(2N|\alpha|^2|\beta|^2)}}{\sqrt{2\pi N|\alpha|^2|\beta|^2}} e^{ig(2l-N)t}, \tag{3.20}$$

that is, $z(t)$ is the Fourier transform of an (approximately) Gaussian distribution and is therefore itself (approximately) Gaussian.

Detailed model calculations, in which the environment is typically represented by a more sophisticated model consisting of a collection of harmonic oscillators (Caldeira and Leggett, 1983; Hu *et al.*, 1992; Joos *et al.*, 2003; Unruh and Zurek, 1989; Zurek, 2003b; Zurek *et al.*, 1993), have shown that the damping occurs on extremely short decoherence time scales τ_D, which are typically many orders of magnitude shorter than the thermal relaxation. Even microscopic systems such as large molecules are rapidly decohered by the interaction with thermal radiation on a time scale that is much shorter than any practical observation could resolve; for mesoscopic systems such as dust particles, the 3K cosmic microwave background radiation is sufficient to yield strong and immediate decoherence (Joos and Zeh, 1985; Zurek, 1991).

Within τ_D, $|z(t)|$ approaches zero and remains close to zero, fluctuating with an average standard deviation of the random-walk type $\sigma \sim \sqrt{N}$ (Zurek, 1982). However, the multiple periodicity of $z(t)$ implies that coherence, and thus the purity of the reduced density matrix, will reappear after a certain time τ_r, which can be shown to be very long and of the Poincaré type with $\tau_r \sim N!$. For macroscopic environments of realistic but finite sizes, τ_r can exceed the lifetime of the universe (Zurek, 1982), but nevertheless always remains finite.

From a conceptual point of view, recurrence of coherence is of little relevance. The recurrence time could only be infinitely long in the hypothetical case of an infinitely large environment. In this situation, off-diagonal terms in the reduced density matrix would be irreversibly damped and lost in the limit $t \to \infty$, which has sometimes been regarded as describing a physical collapse of the state vector (Hepp, 1972). But the assumption of infinite sizes and times is never realized in nature (Bell, 1975), nor can information ever be truly lost (as achieved by a "true" state vector collapse) through unitary time evolution—full coherence is always retained at all times in the total density matrix $\rho_{\mathcal{SAE}}(t) = |\psi(t)\rangle\langle\psi(t)|$.

We can therefore state the general conclusion that, except for exceptionally well-isolated and carefully prepared microsopic and mesoscopic systems, the interaction of the system with the environment causes the off-diagonal terms of the local density matrix, expressed in the pointer basis and describing the probability distribution of the possible outcomes of a measurement on the system, to become extremely small in a very short period of time, and that this process is irreversible for all practical purposes.

E. Environment-induced superselection

Let us now turn to the second main consequence of the interaction with the environment, namely, the environment-induced selection of stable preferred-basis states. We discussed in Sec. II.C the fact that the quantum-mechanical measurement scheme as represented by Eq. (2.1) does not uniquely define the expansion of the postmeasurement state and thereby leaves open the question of which observable can be considered as having been measured by the apparatus. This situation is changed by the inclusion of the environment states in Eq. (3.5), for the following two reasons:

(1) *Environment-induced superselection of a preferred basis.* The interaction between the apparatus and the environment singles out a set of mutually commuting observables.

(2) *The existence of a tridecompositional uniqueness theorem* (Bub, 1997; Clifton, 1994; Elby and Bub, 1994). If a state $|\psi\rangle$ in a Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ can be decomposed into the diagonal ("Schmidt") form $|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle_1 |\phi_i\rangle_2 |\phi_i\rangle_3$, the expansion is unique provided that the $\{|\phi_i\rangle_1\}$ and $\{|\phi_i\rangle_2\}$ are sets of linearly independent, normalized vectors in $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, and that $\{|\phi_i\rangle_3\}$ is a set of mutually noncollinear normalized vectors in $\mathcal{H}_3$. This can be generalized to an N-decompositional uniqueness theorem, in which $N \geq 3$. Note that it is not always possible to decompose an arbitrary pure state of more than two systems ($N \geq 3$) into the Schmidt form $|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle_1 |\phi_i\rangle_2 \cdots |\phi_i\rangle_N$, but if the decomposition exists, its uniqueness is guaranteed.

The tridecompositional uniqueness theorem ensures that the expansion of the final state in Eq. (3.5) is unique, which fixes the ambiguity in the choice of the set of possible outcomes. It demonstrates that the inclusion of (at least) a third "system" (here referred to as the environment) is necessary to remove the basis ambiguity.

Of course, given any pure state in the composite Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$, the tridecompositional uniqueness theorem neither tells us whether a Schmidt decomposition exists nor specifies the unique expansion itself (provided the decomposition is possible), and since the precise states of the environment are generally not known, an additional criterion is needed that determines what the preferred states will be.

1. Stability criterion and pointer basis

The decoherence program has attempted to define such a criterion based on the interaction with the environment and the idea of robustness and preservation of correlations. The environment thus plays a double role in suggesting a solution to the preferred-basis problem: it selects a preferred pointer basis, and it guarantees its uniqueness via the tridecompositional uniqueness theorem.

In order to motivate the basis superselection approach proposed by the decoherence program, we note that in step (2) of Eq. (3.5) we tacitly assumed that interaction with the environment does not disturb the established correlation between the state of the system, $|s_n\rangle$, and the corresponding pointer state $|a_n\rangle$. This assumption can be viewed as a generalization of the concept of "faithful measurement" to the realistic case in which the environment is included. Faithful measurement in the usual sense concerns step (1), namely, the requirement that the measuring apparatus $\mathcal{A}$ act as a reliable "mirror" of the states of the system $\mathcal{S}$ by forming only correlations of the form $|s_n\rangle|a_n\rangle$ but not $|s_m\rangle|a_n\rangle$ with $m \neq n$. But since any realistic measurement process must include the inevitable coupling of the apparatus to its environment, the measurement could hardly be considered faithful as a whole if the interaction with the environment disturbed the correlations between the system and the apparatus.[9]

It was therefore first suggested by Zurek (1981) that the preferred pointer basis be taken as the basis which "contains a reliable record of the state of the system $\mathcal{S}$" (Zurek, 1981, p. 1519), i.e., the basis in which the system-apparatus correlations $|s_n\rangle|a_n\rangle$ are left undisturbed by the subsequent formation of correlations with the environment (the *stability criterion*). One can then find a sufficient criterion for dynamically stable pointer states that preserve the system–apparatus correlations in spite of the interaction of the apparatus with the environment by requiring all pointer state projection operators $\widehat{P}_n^{(\mathcal{A})} = |a_n\rangle\langle a_n|$ to commute with the apparatus-environment interaction Hamiltonian $\widehat{H}_{\mathcal{A}\mathcal{E}}$,[10]

$$[\widehat{P}_n^{(\mathcal{A})}, \widehat{H}_{\mathcal{A}\mathcal{E}}] = 0 \qquad \text{for all } n. \tag{3.21}$$

This implies that any correlation of the measured system (or any other system, for instance an observer) with the eigenstates of a *preferred apparatus observable*,

$$\widehat{O}_{\mathcal{A}} = \sum_n \lambda_n \widehat{P}_n^{(\mathcal{A})}, \tag{3.22}$$

is preserved, and that the states of the environment reliably mirror the pointer states $\widehat{P}_n^{(\mathcal{A})}$. In this case, the environment can be regarded as carrying out a nondemolition measurement on the apparatus. The commutativity requirement, Eq. (3.21), is obviously fulfilled if $\widehat{H}_{\mathcal{A}\mathcal{E}}$ is a function of $\widehat{O}_{\mathcal{A}}$, $\widehat{H}_{\mathcal{A}\mathcal{E}} = \widehat{H}_{\mathcal{A}\mathcal{E}}(\widehat{O}_{\mathcal{A}})$. Conversely, system-apparatus correlations in which the states of the apparatus are not eigenstates of an observable that commutes with $\widehat{H}_{\mathcal{A}\mathcal{E}}$ will in general be rapidly destroyed by the interaction.

Put the other way around, this implies that the environment determines through the form of the interaction Hamiltonian $\widehat{H}_{\mathcal{A}\mathcal{E}}$, a preferred apparatus observable $\widehat{O}_{\mathcal{A}}$, Eq. (3.22), and thereby also the states of the system that are measured by the apparatus, that is, reliably recorded through the formation of dynamically stable quantum correlations. The tridecompositional uniqueness theorem then guarantees the uniqueness of the expansion of the final state $|\psi\rangle = \sum_n c_n|s_n\rangle|a_n\rangle|e_n\rangle$ (where no constraints on the c_n have to be imposed) and thereby the uniqueness of the preferred pointer basis.

Other criteria similar to the commutativity requirement, Eq. (3.21), have been suggested for the selection of the preferred pointer basis because it turns out that in realistic cases the simple relation of Eq. (3.21) can usually only be fulfilled approximately (Zurek, 1993; Zurek *et al.*, 1993). More general criteria, for example, have been based on the von Neumann entropy $-\mathrm{Tr}\rho_\Psi^2(t)\ln\rho_\Psi^2(t)$, or the purity $\mathrm{Tr}\rho_\Psi^2(t)$, with the goal of finding the most robust states or the states which become least entangled with the environment in the course of the evolution (Zurek, 1993, 1998, 2003b; Zurek *et al.*, 1993). Pointer states are obtained by extremizing the measure (i.e., minimizing entropy, or maximizing purity, etc.) over the initial state $|\Psi\rangle$ and requiring the resulting states to be robust when varying the time t. Application of this method leads to a ranking of the possible pointer states with respect to their "classicality," i.e., their robustness with respect to interaction with the environment, and thus allows for the selection of preferred pointer basis in terms of the "most classical" pointer states (the "predictability sieve"; see Zurek, 1993; Zurek *et al.*, 1993). Although the proposed criteria differ somewhat and other meaningful criteria are likely to be suggested in the future, it is hoped that in the macrosopic limit the resulting stable pointer states obtained from different criteria will turn out to be very similar (Zurek, 2003b). For some toy models (in particular, for harmonic-oscillator models that lead to coherent states as pointer states), this has already been verified explicitly (see, for example, Diósi and Kiefer, 2000; Eisert, 2004; Joos *et al.*, 2003; Kübler and Zeh, 1973; Zurek, 1993).

2. Selection of quasiclassical properties

System-environment interaction Hamiltonians frequently describe a scattering process of surrounding particles (photons, air molecules, etc.) interacting with the system under study. Since the force laws describing such processes typically depend on some power of distance (such as $\propto r^{-2}$ in Newton's or Coulomb's force law), the interaction Hamiltonian will usually commute with the position basis, such that, according the commutativity requirement of Eq. (3.21), the preferred basis will be in position space. The fact that position is frequently the determinate property of our experience can then be explained by referring to the dependence of most interactions on distance (Zurek, 1981, 1982, 1991).

This holds, in particular, for mesoscopic and macroscopic systems, as demonstrated, for instance, by the pioneering study of Joos and Zeh (1985), in which sur-

[9] For fundamental limitations on the precision of von Neumann measurements of operators that do not commute with a globally conserved quantity, see the Wigner-Araki-Yanase theorem (Araki and Yanase, 1960; Wigner, 1952).

[10] For simplicity, we assume here that the environment $\mathcal{E}$ interacts directly only with the apparatus $\mathcal{A}$, but not with the system $\mathcal{S}$.

rounding photons and air molecules are shown to continuously "measure" the spatial structure of dust particles, leading to rapid decoherence into an apparent (improper) mixture of wave packets that are sharply peaked in position space. Similar results sometimes even hold for microscopic systems (usually found in energy eigenstates; see below) when they occur in distinct spatial structures that couple strongly to the surrounding medium. For instance, chiral molecules such as sugar are always observed to be in chirality eigenstates (left-handed and right-handed) which are superpositions of different energy eigenstates (Harris and Stodolsky, 1981; Zeh, 2000). This is explained by the fact that the spatial structure of these molecules is continuously "monitored" by the environment, for example, through the scattering of air molecules, which gives rise to a much stronger coupling than could typically be achieved by a measuring device that was intended to measure, say, parity or energy; furthermore, any attempt to prepare such molecules in energy eigenstates would lead to immediate decoherence into environmentally stable ("dynamically robust") chirality eigenstates, thus selecting position as the preferred basis.

On the other hand, it is well known that many systems, especially in the microsopic domain, are typically found in energy eigenstates, even if the interaction Hamiltonian depends on a different observable than energy, e.g., position. Paz and Zurek (1999) have shown that this situation arises when the predominant frequencies present in the environment are significantly lower than the intrinsic frequencies of the system, that is, when the separation between the energy states of the system is greater than the largest energies available in the environment. Then, the environment will only be able to monitor quantities that are constants of motion. In the case of nondegeneracy, this will be energy, thus leading to the environment-induced superselection of energy eigenstates for the system.

Another example of environment-induced superselection that has been studied is related to the fact that only eigenstates of the charge operator are observed, but never superpositions of different charges. The existence of the corresponding superselection rules was first only postulated (Wick *et al.*, 1952, 1970), but could subsequently be explained in the framework of decoherence by referring to the interaction of the charge with its own Coulomb (far) field, which takes the role of an "environment," leading to immediate decoherence of charge superpositions into an apparent mixture of charge eigenstates (Giulini, 2000; Giulini *et al.*, 1995).

In general, three different cases have typically been distinguished (for example, in Paz and Zurek, 1999) for the kind of pointer observable emerging from an interaction with the environment, depending on the relative strengths of the system's self-Hamiltonian $\widehat{H}_{\mathcal{S}}$ and of the system-environment interaction Hamiltonian $\widehat{H}_{\mathcal{SE}}$:

(1) When the dynamics of the system are dominated by $\widehat{H}_{\mathcal{SE}}$, i.e., the interaction with the environment, the pointer states will be eigenstates of $\widehat{H}_{\mathcal{SE}}$ (and thus typically eigenstates of position). This case corresponds to the typical quantum measurement setting; see, for example, the model of Zurek (1981, 1982), which is outlined in Sec. III.D.2 above.

(2) When the interaction with the environment is weak and $\widehat{H}_{\mathcal{S}}$ dominates the evolution of the system (that is, when the environment is "slow" in the above sense), a case that frequently occurs in the microscopic domain, pointer states will arise that are energy eigenstates of $\widehat{H}_{\mathcal{S}}$ (Paz and Zurek, 1999).

(3) In the intermediate case, when the evolution of the system is governed by $\widehat{H}_{\mathcal{SE}}$ and $\widehat{H}_{\mathcal{S}}$ in roughly equal strength, the resulting preferred states will represent a "compromise" between the first two cases; for instance, the frequently studied model of quantum Brownian motion has shown the emergence of pointer states localized in phase space, i.e., in both position and momentum (Eisert, 2004; Joos *et al.*, 2003; Unruh and Zurek, 1989; Zurek, 2003b; Zurek *et al.*, 1993).

3. Implications for the preferred-basis problem

The decoherence program proposes that the preferred basis be selected by the requirement that correlations be preserved in spite of the interaction with the environment, and thus be chosen through the form of the system-environment interaction Hamiltonian. This seems certainly reasonable, since only such "robust" states will in general be observable—and, after all, we solely seek an explanation for our experience (see the discussion in Sec. II.B.3). Although only particular examples have been studied (for a survey and references, see, for example, Blanchard *et al.*, 2000; Joos *et al.*, 2003; Zurek, 2003b), the results thus far suggest that the selected properties are in agreement with our observation: for mesoscopic and macroscopic objects the distance-dependent scattering interaction with surrounding air molecules, photons, etc., will in general give rise to immediate decoherence into spatially localized wave packets and thus select position as the preferred basis. On the other hand, when the environment is comparably "slow," as is frequently the case for microsopic systems, environment-induced superselection will typically yield energy eigenstates as the preferred states.

The clear merit of the approach of environment-induced superselection lies in the fact that the preferred basis is not chosen in an *ad hoc* manner simply to make our measurement records determinate or to match our experience of which physical quantities are usually perceived as determinate (for example, position). Instead the selection is motivated on physical, observer-free grounds, that is, through the system-environment interaction Hamiltonian. The vast space of possible quantum-mechanical superpositions is reduced so much because the laws governing physical interactions depend only on a few physical quantities (position, momentum, charge, and the like), and the fact that precisely these are the properties that appear determinate to us is explained by the dependence of the preferred basis on the form of the interaction. The appearance of "classicality" is therefore grounded in the structure of the physical laws—certainly a highly satisfying and reasonable approach.

The above argument in favor of the approach of environment-induced superselection could, of course, be considered as inadequate on a fundamental level: All physical laws are discovered and formulated by us, so they can contain only the determinate quantities of our experience. These are the only quantities we can perceive and thus include in a physical law. Thus the derivation of determinacy from the structure of our physical laws might seem circular. However, we argue again that it suffices to demand a subjective solution to the preferred-basis problem—that is, to provide an answer to the question of why we perceive only such a small subset of properties as determinate, not whether there really are determinate properties (on an ontological level) and what they are (cf. the remarks in Sec. II.B.3).

We might also worry about the generality of this approach. One would need to show that any such environment-induced superselection leads,in fact, to precisely those properties that appear determinate to us. But this would require precise knowledge of the system and the interaction Hamiltonian. For simple toy models, the relevant Hamiltonians can be written down explicitly. In more complicated and realistic cases, this will in general be very difficult, if not impossible, since the form of the Hamiltonian will depend on the particular system or apparatus and the monitoring environment under consideration, where, in addition, the environment is not only difficult to define precisely, but also constantly changing, uncontrollable, and, in essence, infinitely large.

But the situation is not as hopeless as it might sound, since we know that the interaction Hamiltonian will, in general, be based on the set of known physical laws which, in turn, employ only a relatively small number of physical quantities. So as long as we assume the stability criterion and consider the set of known physical quantities as complete, we can automatically anticipate that the preferred basis will be a member of this set. The remaining, yet very relevant, question is then which subset of these properties will be chosen in a specific physical situation (for example, will the system preferably be found in an eigenstate of energy or position?), and to what extent this will match the experimental evidence. To give an answer, one will usually need a more detailed knowledge of the interaction Hamiltonian and of its relative strength with respect to the self-Hamiltonian of the system in order to verify the approach. Besides, as mentioned in Sec. III.E, there exist other criteria than the commutativity requirement, and whether they all lead to the same determinate properties is a question that has not yet been fully explored.

Finally, a fundamental conceptual difficulty of the decoherence-based approach to the preferred-basis problem is the lack of a general criterion for what defines the systems and the "unobserved" degrees of freedom of the environment (see the discussion in Sec. III.A). While in many laboratory-type situations, the division into system and environment might seem straightforward, it is not clear *a priori* how quasiclassical observables can be defined through environment-induced superselection on a larger and more general scale, when larger parts of the universe are considered where the split into subsystems is not suggested by some specific system-apparatus-surroundings setup.

To summarize, environment-induced superselection of a preferred basis (i) proposes an explanation for why a particular pointer basis gets chosen at all—by arguing that it is only the pointer basis that leads to stable, and thus perceivable, records when the interaction of the apparatus with the environment is taken into account; and (ii) argues that the preferred basis will correspond to a subset of the set of the determinate properties of our experience, since the governing interaction Hamiltonian will depend solely on these quantities. But it does not tell us precisely what the pointer basis will be in any given physical situation, since it will usually be hardly possible to write down explicitly the relevant interaction Hamiltonian in realistic cases. This also means that it will be difficult to argue that any proposed criterion based on the interaction with the environment will always and in all generality lead to exactly those properties that we perceive as determinate.

More work remains to be done, therefore, to fully explore the general validity and applicability of the approach of environment-induced superselection. But since the results obtained thus far from toy models have been in promising agreement with empirical data, there is little reason to doubt that the decoherence program has proposed a very valuable criterion for explaining the emergence of preferred states and their robustness. The fact that the approach is derived from physical principles should be counted additionally in its favor.

4. Pointer basis vs instantaneous Schmidt states

The so-called Schmidt basis, obtained by diagonalizing the (reduced) density matrix of the system at each instant of time, has been frequently studied with respect to its ability to yield a preferred basis (see, for example, Albrecht, 1992, 1993; Zeh, 1973), having led some to consider the Schmidt basis states as describing "instantaneous pointer states" (Albrecht, 1992). However, as it has been emphasized (for example, by Zurek, 1993), any density matrix is diagonal in some basis, and this basis will in general not play any special interpretive role. Pointer states that are supposed to correspond to quasiclassical stable observables must be derived from an explicit criterion for classicality (typically, the stability criterion); the simple mathematical diagonalization procedure of the instantaneous density matrix will generally not suffice to determine a quasiclassical pointer basis (see the studies by Barvinsky and Kamenshchik, 1995; Kent and McElwaine, 1997).

In a more refined method, one refrains from computing instantaneous Schmidt states and instead allows for a characteristic decoherence time τ_D to pass during which the reduced density matrix decoheres (a process that can be described by an appropriate master equation) and becomes approximately diagonal in the stable pointer basis, the basis that is selected by the stability criterion. Schmidt states are then calculated by diagonalizing the decohered density matrix. Since decoherence usually leads to rapid diagonality of the reduced density matrix in the stability-selected pointer basis to a very good approximation, the resulting Schmidt states are typically very similar to the pointer basis except when the pointer states are very nearly degenerate. The latter situation is readily illustrated by considering the approximately diagonalized decohered density matrix

$$\rho = \begin{pmatrix} 1/2+\delta & \omega^* \\ \omega & 1/2-\delta \end{pmatrix}, \tag{3.23}$$

where $|\omega| \ll 1$ (strong decoherence) and $\delta \ll 1$ (near-degeneracy; Albrecht, 1993). If decoherence led to exact diagonality, $\omega = 0$, the eigenstates would be, for any fixed value of δ, proportional to $(0,1)$ and $(1,0)$ (corresponding to the "ideal" pointer states). However, for fixed $\omega > 0$ (approximate diagonality) and $\delta \to 0$ (degeneracy), the eigenstates become proportional to $(\pm|\omega|/\omega, 1)$, which implies that, in the case of degeneracy, the Schmidt decomposition of the reduced density matrix can yield preferred states that are very different from the stable pointer states, even if the decohered, rather than the instantaneous, reduced density matrix is diagonalized.

In summary, it is important to emphasize that stability (or a similar criterion) is the relevant requirement for the emergence of a preferred quasiclassical basis, which cannot, in general, be achieved by simply diagonalizing the instantaneous reduced density matrix. However, the eigenstates of the decohered reduced density matrix will, in many situations, approximate the quasiclassical stable pointer states well, especially when these pointer states are sufficiently nondegenerate.

F. Envariance, quantum probabilities, and the Born rule

In the following, we shall review an interesting and promising approach introduced recently by Zurek (2003b,c, 2004a,b) that aims to explain the emergence of quantum probabilities and to deduce the Born rule based on a mechanism termed "environment-assisted invariance," or "envariance" for short, a particular symmetry property of entangled quantum states. The original exposition of Zurek (2003b) was followed up by several articles by other authors, who assessed the approach, pointed out more clearly the assumptions entering into the derivation, and presented variants of the proof (Barnum, 2003; Mohrhoff, 2004; Schlosshauer and Fine, 2005). An expanded treatment of envariance and quantum probabilities that addresses some of the issues discussed in these papers and that offers an interesting outlook on further implications of envariance can be found in Zurek (2004a). In our outline of the theory of envariance, we shall follow this most recent treatment, as it spells out the derivation and the required assumptions more explicitly and in greater detail and clarity than in Zurek's earlier (2003b; 2003c; 2004b) papers (cf. also the remarks of Schlosshauer and Fine, 2005).

We include a discussion of Zurek's proposal here for two reasons. First, the derivation is based on the inclusion of an environment $\mathcal{E}$, entangled with the system $\mathcal{S}$ of interest to which probabilities of measurement outcomes are to be assigned, and thus it matches well the spirit of the decoherence program. Second, and more importantly, despite the contributions of decoherence to explaining the emergence of subjective classicality from quantum mechanics, a consistent derivation of classicality (including a motivation for some of the axioms of quantum mechanics, as suggested by Zurek, 2003b) requires the separate derivation of the Born rule. The decoherence program relies heavily on the concept of reduced density matrices and the related formalism and interpretation of the trace operation, see Eq. (3.6), which *presuppose* Born's rule. Therefore decoherence itself cannot be used to derive the Born rule (as was tried, for example, by Deutsch, 1999 and Zurek, 1998) since otherwise the argument would be rendered circular (Zeh, 1997; Zurek, 2003b).

There have been various attempts in the past to replace the postulate of the Born rule by a derivation. Gleason's (1957) theorem has shown that if one imposes the condition that for any orthonormal basis of a given Hilbert space the sum of the probabilities associated with each basis vector must add up to one, the Born rule is the only possibility for the calculation of probabilities. However, Gleason's proof provides little insight into the physical meaning of the Born probabilities, and therefore various other attempts, typically based on a relative frequencies approach (i.e., on a counting argument), have been made towards a derivation of the Born rule in a no-collapse (and usually relative-state) setting (see, for example, Deutsch, 1999; DeWitt, 1971; DeWitt and Graham, 1973; Everett, 1957; Farhi *et al.*, 1989; Geroch, 1984; Graham, 1973; Hartle, 1968). However, it was pointed out that these approaches fail due to the use of circular arguments (Barnum *et al.*, 2000; Kent, 1990; Squires, 1990; Stein, 1984); cf. also Wallace (2003b) and Saunders (2002).

Zurek's recently developed theory of envariance provides a promising new strategy for deriving, given certain assumptions, the Born rule in a manner that avoids the circularities of the earlier approaches. We shall outline the concept of envariance in the following and show how it can lead to Born's rule.

1. Environment-assisted invariance

Zurek introduces his definition of envariance as follows. Consider a composite state $|\psi_{\mathcal{SE}}\rangle$ (where, as usual, $\mathcal{S}$ refers to the "system" and $\mathcal{E}$ to some "environment") in a Hilbert space given by the tensor product $\mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{E}$, and a pair of unitary transformations $\widehat{U}_\mathcal{S} = \widehat{u}_\mathcal{S} \otimes \widehat{I}_\mathcal{E}$ and $\widehat{U}_\mathcal{E} = \widehat{I}_\mathcal{S} \otimes \widehat{u}_\mathcal{E}$ acting on $\mathcal{S}$ and $\mathcal{E}$, respectively. If $|\psi_{\mathcal{SE}}\rangle$ is invariant under the combined application of $\widehat{U}_\mathcal{S}$ and $\widehat{U}_\mathcal{E}$,

$$\widehat{U}_\mathcal{E}(\widehat{U}_\mathcal{S}|\psi_{\mathcal{SE}}\rangle) = |\psi_{\mathcal{SE}}\rangle, \tag{3.24}$$

$|\psi_{\mathcal{SE}}\rangle$ is called *envariant under* $\widehat{u}_\mathcal{S}$. In other words, the change in $|\psi_{\mathcal{SE}}\rangle$ induced by acting on $\mathcal{S}$ via $\widehat{U}_\mathcal{S}$ can be undone by acting on $\mathcal{E}$ via $\widehat{U}_\mathcal{E}$. Note that envariance is a distinctly quantum feature, absent from pure classical states, and a consequence of quantum entanglement.

The main argument of Zurek's derivation is based on a study of a composite pure state in the diagonal Schmidt decomposition

$$|\psi_{\mathcal{SE}}\rangle = \frac{1}{\sqrt{2}}\big(e^{i\varphi_1}|s_1\rangle|e_1\rangle + e^{i\varphi_2}|s_1\rangle|e_1\rangle\big), \tag{3.25}$$

where the $\{|s_k\rangle\}$ and $\{|e_k\rangle\}$ are sets of orthonormal basis vectors that span the Hilbert spaces $\mathcal{H}_\mathcal{S}$ and $\mathcal{H}_\mathcal{E}$, respectively. The case of higher-dimensional state spaces can be treated similarly, and a generalization to expansion coefficients of different magnitudes can be made by application of a standard counting argument (Zurek, 2003c, 2004a). The Schmidt states $|s_k\rangle$ are identified with the outcomes, or "events" (Zurek, 2004b, p. 12), to which probabilities are to be assigned.

Zurek now states three simple assumptions, called "facts" (Zurek, 2004a, p. 4; see also the discussion in Schlosshauer and Fine, 2005):

(A1) A unitary transformation of the form $\cdots \otimes \widehat{I}_\mathcal{S} \otimes \cdots$ does not alter the state of $\mathcal{S}$.

(A2) All measurable properties of $\mathcal{S}$, including probabilities of outcomes of measurements on $\mathcal{S}$, are fully determined by the state of $\mathcal{S}$.

(A3) The state of $\mathcal{S}$ is completely specified by the global composite state vector $|\psi_{\mathcal{SE}}\rangle$.

Given these assumptions, one can show that the state of $\mathcal{S}$ and any measurable properties of $\mathcal{S}$ cannot be affected by envariant transformations. The proof goes as follows. The effect of an envariant transformation $\widehat{u}_\mathcal{S} \otimes \widehat{I}_\mathcal{E}$ acting on $|\psi_{\mathcal{SE}}\rangle$ can be undone by a corresponding "countertransformation" $\widehat{I}_\mathcal{S} \otimes \widehat{u}_\mathcal{E}$ that restores the original state vector $|\psi_{\mathcal{SE}}\rangle$. Since it follows from (A1) that the latter transformation has left the state of $\mathcal{S}$ unchanged, but (A3) implies that the final state of $\mathcal{S}$ (after the transformation and countertransformation) is identical to the initial state of $\mathcal{S}$, the first transformation $\widehat{u}_\mathcal{S} \otimes \widehat{I}_\mathcal{E}$ cannot have altered the state of $\mathcal{S}$ either. Thus, using assumption (A2), it follows that an envariant transformation $\widehat{u}_\mathcal{S} \otimes \widehat{I}_\mathcal{E}$ acting on $|\psi_{\mathcal{SE}}\rangle$ leaves any measurable properties of $\mathcal{S}$ unchanged, in particular the probabilities associated with outcomes of measurements performed on $\mathcal{S}$.

Let us now consider two different envariant transformations: A *phase transformation* of the form

$$\widehat{u}_\mathcal{S}(\xi_1, \xi_2) = e^{i\xi_1}|s_1\rangle\langle s_1| + e^{i\xi_2}|s_2\rangle\langle s_2| \tag{3.26}$$

that changes the phases associated with the Schmidt product states $|s_k\rangle|e_k\rangle$ in Eq. (3.25), and a *swap transformation*

$$\widehat{u}_\mathcal{S}(1 \leftrightarrow 2) = e^{i\xi_{12}}|s_1\rangle\langle s_2| + e^{i\xi_{21}}|s_2\rangle\langle s_1| \tag{3.27}$$

that exchanges the pairing of the $|s_k\rangle$ with the $|e_l\rangle$. Based on the assumptions (A1)–(A3) mentioned above, envariance of $|\psi_{\mathcal{SE}}\rangle$ under these transformations means that measurable properties of $\mathcal{S}$ cannot depend on the phases φ_k in the Schmidt expansion of $|\psi_{\mathcal{SE}}\rangle$, Eq. (3.25). Similarily, it follows that a swap $\widehat{u}_\mathcal{S}(1 \leftrightarrow 2)$ leaves the state of $\mathcal{S}$ unchanged, and that the consequences of the swap cannot be detected by any measurement that pertains to $\mathcal{S}$ alone.

2. Deducing the Born rule

Together with an additional assumption, this result can then be used to show that the probabilities of the "outcomes" $|s_k\rangle$ appearing in the Schmidt decomposition of $|\psi_{\mathcal{SE}}\rangle$ must be equal, thus arriving at Born's rule for the special case of a state-vector expansion with coefficients of equal magnitude. Zurek (2004a) offers three possibilities for such an assumption. Here we shall limit our discussion to one of these possible assumptions (see also the comments in Schlosshauer and Fine, 2005):

(A4) The Schmidt product states $|s_k\rangle|e_k\rangle$ appearing in the state-vector expansion of $|\psi_{\mathcal{SE}}\rangle$ imply a direct and perfect correlation of the measurement outcomes associated with the $|s_k\rangle$ and $|e_k\rangle$. That is, if an observable $\widehat{O}_\mathcal{S} = \sum s_{kl}|s_k\rangle\langle s_l|$ is measured on $\mathcal{S}$ and $|s_k\rangle$ is obtained, a subsequent measurement of $\widehat{O}_\mathcal{E} = \sum e_{kl}|e_k\rangle\langle e_l|$ on $\mathcal{E}$ will yield $|e_k\rangle$ with certainty (i.e., with probability equal to one).

This assumption explicitly introduces a probability concept into the derivation. (Similarly, the two other possible assumptions suggested by Zurek establish a connection between the state of $\mathcal{S}$ and probabilities of outcomes of measurements on $\mathcal{S}$.)

Then, denoting the probability for the outcome $|s_k\rangle$ by $p(|s_k\rangle, |\psi_{\mathcal{SE}}\rangle)$ when the composite system $\mathcal{SE}$ is described by the state vector $|\psi_{\mathcal{SE}}\rangle$, this assumption implies that

$$p(|s_k\rangle; |\psi_{\mathcal{SE}}\rangle) = p(|e_k\rangle; |\psi_{\mathcal{SE}}\rangle). \tag{3.28}$$

After acting on $|\psi_{\mathcal{SE}}\rangle$ with the envariant swap transformation $\widehat{U}_{\mathcal{S}} = \widehat{u}_{\mathcal{S}}(1 \leftrightarrow 2) \otimes \widehat{I}_{\mathcal{E}}$ [see Eq. (3.27)] and using assumption (A4) again, we get

$$\begin{aligned} p(|s_1\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_2\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle), \\ p(|s_2\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_1\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle). \end{aligned} \tag{3.29}$$

Now, when a "counterswap" $\widehat{U}_{\mathcal{E}} = \widehat{I}_{\mathcal{S}} \otimes u_{\mathcal{E}}(1 \leftrightarrow 2)$ is applied to $|\psi_{\mathcal{SE}}\rangle$, the original state vector $|\psi_{\mathcal{SE}}\rangle$ is restored, i.e., $\widehat{U}_{\mathcal{E}}(\widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = |\psi_{\mathcal{SE}}\rangle$. It then follows from assumptions (A2) and (A3) listed above that

$$p(|s_k\rangle; \widehat{U}_{\mathcal{E}}\widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = p(|s_k\rangle; |\psi_{\mathcal{SE}}\rangle). \tag{3.30}$$

Furthermore, assumptions (A1) and (A2) imply that the first and second swap cannot have affected the measurable properties of $\mathcal{E}$ and $\mathcal{S}$, respectively, particularly not the probabilities for outcomes of measurements on $\mathcal{E}$ ($\mathcal{S}$),

$$\begin{aligned} p(|s_k\rangle; \widehat{U}_{\mathcal{E}}\widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|s_k\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle), \\ p(|e_k\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_k\rangle; |\psi_{\mathcal{SE}}\rangle). \end{aligned} \tag{3.31}$$

Combining Eqs. (3.28)–(3.31) yields

$$\begin{aligned} p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle) &\overset{(3.30)}{=} p(|s_1\rangle; \widehat{U}_{\mathcal{E}}\widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\overset{(3.31)}{=} p(|s_1\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\overset{(3.29)}{=} p(|e_2\rangle; \widehat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\overset{(3.31)}{=} p(|e_2\rangle; |\psi_{\mathcal{SE}}\rangle) \\ &\overset{(3.28)}{=} p(|s_2\rangle; |\psi_{\mathcal{SE}}\rangle), \end{aligned} \tag{3.32}$$

which establishes the desired result $p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle) = p(|s_2\rangle; |\psi_{\mathcal{SE}}\rangle)$. The general case of unequal coefficients in the Schmidt decomposition of $|\psi_{\mathcal{SE}}\rangle$ can then be treated by means of a simple counting method (Zurek, 2003c, 2004a), leading to Born's rule for probabilities that are rational numbers. Using a continuity argument, this result can be further generalized to include probabilities that cannot be expressed as rational numbers (Zurek, 2004a).

3. Summary and outlook

If one grants the stated assumptions, Zurek's development of the theory of envariance offers a novel and promising way of deducing Born's rule in a noncircular manner. Compared to the relatively well-studied field of decoherence, envariance and its consequences have only begun to be explored. In this review, we have focused on envariance in the context of a derivation of the Born rule, but other far-reaching implications of envariance have recently been suggested by Zurek (2004a). For example, envariance could also account for the emergence of an environment-selected preferred basis (that is, for environment-induced superselection) without an appeal to the trace operation or to reduced density matrices. This could open up the possibility of a redevelopment of the decoherence program based on fundamental quantum-mechanical principles that do not require one to presuppose the Born rule; this also might shed new light, for example, on the interpretation of reduced density matrices that has led to much controversy in discussions of decoherence (see Sec. III.B). As of now, the development of such ideas is at a very early stage, but we can expect further interesting results derived from envariance in the near future.

IV. THE ROLE OF DECOHERENCE IN INTERPRETATIONS OF QUANTUM MECHANICS

It was not until the early 1970s that the importance of the interaction of physical systems with their environments for a realistic quantum-mechanical description of these systems was realized and a proper viewpoint on such interactions was established (Zeh, 1970, 1973). It took another decade for the first concise formulation of the theory of decoherence (Zurek, 1981, 1982) to be worked out and for numerical studies to be made that showed the ubiquity and effectiveness of decoherence effects (Joos and Zeh, 1985). Of course, by that time, several interpretive approaches to quantum mechanics had already been established, for example, Everett-style relative-state interpretations (Everett, 1957), the concept of modal interpretations introduced by van Fraassen (1973, 1991), and the pilot-wave theory of de Broglie and Bohm (Bohm, 1952).

When the relevance of decoherence effects was recognized by (parts of) the scientific community, decoherence provided a motivation for a fresh look at the existing interpretations and for the introduction of changes and extensions to these interpretations, as well as for new interpretations. Some of the central questions in this context were, and still are, the following:

1. Can decoherence by itself solve certain foundational issues at least for all practical purposes, such as to make certain interpretive additives superfluous? What, then, are the crucial remaining foundational problems?

2. Can decoherence protect an interpretation from empirical disproof?

3. Conversely, can decoherence provide a mechanism to exclude an interpretive strategy as incompatible with quantum mechanics and/or as empirically inadequate?

4. Can decoherence physically motivate some of the assumptions on which an interpretation is based and give them a more precise meaning?

5. Can decoherence serve as an amalgam that would unify and simplify a spectrum of different interpretations?

These and other questions have been widely discussed, both in the context of particular interpretations and with respect to the general implications of decoherence for any interpretation of quantum mechanics. In particular, interpretations that uphold the universal validity of the unitary Schrödinger time evolution, most notably relative-state and modal interpretations, have frequently incorporated environment-induced superselection of a preferred basis and decoherence into their framework. It is the purpose of this section to critically investigate the implications of decoherence for the existing interpretations of quantum mechanics, with particular attention to the questions outlined above.

A. General implications of decoherence for interpretations

When measurements are understood as ubiquitous interactions that lead to the formation of quantum correlations, the selection of a preferred basis becomes in most cases a fundamental requirement. This also corresponds, in general, to the question of what properties are being ascribed to systems (or worlds, minds, etc.). Thus the preferred-basis problem is at the heart of any interpretation of quantum mechanics. Some of the difficulties that must be faced in solving the preferred-basis problem are

(i) to decide whether the selection of any preferred basis (or quantity or property) is justified at all or only an artefact of our subjective experience;

(ii) if we decide on (i) in the positive, to select those determinate quantity or quantities (what appears determinate to us does not need to be appear determinate to other kinds of observers, nor does it need to be the "true" determinate property);

(iii) to avoid any *ad hoc* character of the choice and any possible empirical inadequacy or inconsistency with the confirmed predictions of quantum mechanics;

(iv) if a multitude of quantities is selected that apply differently among different systems, to be able to formulate specific rules that specify the determinate quantity or quantities under every circumstance;

(v) to ensure that the basis is chosen such that if the system is embedded in a larger (composite) system, the principle of property composition holds, i.e., the property selected by the basis of the original system should also persist when the system is considered as part of a larger composite system.[11]

The hope is then that environment-induced superselection of a preferred basis can provide a universal mechanism that fulfills the above criteria and solves the preferred-basis problem on strictly physical grounds.

A popular reading of the decoherence program typically goes as follows. First, the interaction of the system with the environment selects a preferred basis, i.e., a particular set of quasiclassical robust states that commute, at least approximately, with the Hamiltonian governing the system–environment interaction. Since the form of the interaction Hamiltonians usually depends on familiar "classical" quantities, the preferred states will typically also correspond to the small set of "classical" properties. Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as an explanation of the appearance to a local observer of a "classical" world of determinate, "objective" (in the sense of being robust) properties. The tempting interpretation of these achievements is then to conclude that this accounts for the observation of unique (via environment-induced superselection) and definite (via decoherence) pointer states at the end of the measurement, and the measurement problem appears to be solved, at least for all practical purposes.

However, the crucial difficulty in the above reasoning is justifying the second step: How is one to interpret the local suppression of interference in spite of the fact that full coherence is retained in the total state that describes the system-environment combination? While the local destruction of interference allows one to infer the emergence of an (improper) ensemble of individually localized components of the wave function, one still needs to impose an interpretive framework that explains why only one of the localized states is realized and/or perceived. This has been done in various interpretations of quantum mechanics, typically on the basis of the decohered reduced density matrix to ensure consistency with the predictions of the Schrödinger dynamics and thus empirical adequacy.

In this context, one might raise the question whether retention of full coherence in the composite state of the system-environment combination could ever lead to empirical conflicts with the ascription of definite values to (mesoscopic and macroscopic) systems in some decoherence-based interpretive approach. After all, one could think of enlarging the system so as to include the environment in such a way that measurements could now actually reveal the persisting quantum coherence even on a macroscopic level. However, Zurek (1982) asserted that such measurements would be impossible to carry out in practice, a statement that was supported by a simple model calculation by Omnès (1992, p. 356) for a body with a macrosopic number (10^{24}) of degrees of freedom.

[11] This is a problem encountered in some modal interpretations (see Clifton, 1996).

B. The standard and the Copenhagen interpretation

As is well known, the standard interpretation ("orthodox" quantum mechanics) postulates that every measurement induces a discontinuous break in the unitary time evolution of the state through the collapse of the total wave function onto one of its terms in the state-vector expansion (uniquely determined by the eigenbasis of the measured observable), which selects a single term in the superposition as representing the outcome. The nature of the collapse is not at all explained, and thus the definition of measurement remains unclear. Macroscopic superpositions are not *a priori* forbidden, but are never observed since any observation would entail a measurementlike interaction. In the following, we shall also consider a "Copenhagen" variant of the standard interpretation, which adds an additional key element, postulating the necessity of classical concepts in order to describe quantum phenomena, including measurements.

1. The problem of definite outcomes

The interpretive rule of orthodox quantum mechanics that tells us when we can speak of outcomes is given by the e-e link.[12] This is an "objective" criterion since it allows us to infer the existence of a definite state of the system to which a value of a physical quantity can be ascribed. Within this interpretive framework (and without presuming the collapse postulate) decoherence cannot solve the problem of outcomes: Phase coherence between macroscopically different pointer states is preserved in the state that includes the environment, and we can always enlarge the system so as to include (at least parts of) the environment. In other words, the superposition of different pointer positions still exists, coherence is only "*delocalized* into the larger system" (Kiefer and Joos, 1999, p. 5), that is, into the environment—or, as Joos and Zeh (1985, p. 224) put it, "the interference terms still exist, but they are not *there*"—and the process of decoherence could in principle always be reversed. Therefore, if we assume the orthodox e-e link to establish the existence of determinate values of physical quantities, decoherence cannot ensure that the measuring device actually ever is in a definite pointer state (unless, of course, the system is initially in an eigenstate of the observable), or that measurements have outcomes at all. Much of the general criticism directed against decoherence with respect to its ability to solve the measurement problem (at least in the context of the standard interpretation) has been centered on this argument.

[12] It is not particularly relevant for the subsequent discussion whether the e-e link is assumed in its "exact" form, i.e., requiring the exact eigenstates of an observable, or a "fuzzy" form that allows the ascription of definiteness based on only approximate eigenstates or on wave functions with (tiny) "tails."

Note that, with respect to the global postmeasurement state vector, given by the final step in Eq. (3.5), the interaction with the environment has only led to additional entanglement. it has not transformed the state vector in any way, since the rapidly increasing orthogonality of the states of the environment associated with the different pointer positions has not influenced the state description at all. In brief, the entanglement brought about by interaction with the environment could even be considered as making the measurement problem worse. Bacciagaluppi (2003a, Sec. 3.2) puts it like this:

> Intuitively, if the environment is carrying out, without our intervention, lots of approximate position measurements, then the measurement problem ought to apply more widely, also to these spontaneously occurring measurements. (...) The state of the object and the environment could be a superposition of zillions of very well localised terms, each with slightly different positions, and which are collectively spread over a macroscopic distance, even in the case of everyday objects. (...) If everything is in interaction with everything else, everything is entangled with everything else, and that is a worse problem than the entanglement of measuring apparatuses with the measured probes.

Only once we have formed the reduced pure-state density matrix $\widehat{\rho}_{\mathcal{SA}}$, Eq. (3.8), can the orthogonality of the environmental states have an effect; then, $\widehat{\rho}_{\mathcal{SA}}$ dynamically evolves into the improper ensemble $\widehat{\rho}^{d}_{\mathcal{SA}}$ [Eq. (3.9)]. However, as pointed out in our general discussion of reduced density matrices in Sec. III.B, the orthodox rule of interpreting superpositions prohibits regarding the components in the sum of Eq. (3.9) as corresponding to individual well-defined quantum states.

Rather than considering the postdecoherence state of the system (or, more precisely, of the system-apparatus combination $\mathcal{SA}$), we can instead analyze the influence of decoherence on the *expectation values* of observables pertaining to $\mathcal{SA}$; after all, such expectation values are what local observers would measure in order to arrive at conclusions about $\mathcal{SA}$. The diagonalized reduced density matrix, Eq. (3.9), together with the trace relation, Eq. (3.6), implies that, for all practical purposes, the statistics of the system $\mathcal{SA}$ will be indistinguishable from that of a proper mixture (ensemble) by any local observation on $\mathcal{SA}$. That is, given (i) the trace rule $\langle \widehat{O} \rangle = \mathrm{Tr}(\widehat{\rho}\widehat{O})$ and (ii) the interpretation of $\langle \widehat{O} \rangle$ as the expectation value of an observable $\widehat{O}$, the expectation value of any observable $\widehat{O}_{\mathcal{SA}}$ restricted to the local system $\mathcal{SA}$ will be for all practical purposes identical to the expectation value of this observable if $\mathcal{SA}$ had been in one of the states $|s_n\rangle|a_n\rangle$ (as if $\mathcal{SA}$ were described by an ensemble of states). In other words, decoherence has effectively removed any interference terms (such as $|s_m\rangle|a_m\rangle\langle a_n|\langle s_n|$ where $m \neq n$) from the calculation of the trace $\mathrm{Tr}(\widehat{\rho}_{\mathcal{SA}}\widehat{O}_{\mathcal{SA}})$ and thereby from the calculation of the expectation value $\langle \widehat{O}_{\mathcal{SA}} \rangle$. It has therefore been claimed that *formal equivalence*—i.e., the fact that decoherence transforms the reduced density matrix into a form identical to that of a density matrix representing an ensemble of pure states—yields *observational equivalence* in the sense above, namely, the (local) indistiguishability of the expectation values derived from these two types of density matrices via the trace rule.

But we must be careful in interpreting the correspondence between the mathematical formalism (such as the trace rule) and the common terms employed in describing "the world." In quantum mechanics, the identification of the expression "$\mathrm{Tr}(\rho A)$" as the expectation value of a quantity relies on the mathematical fact that, when writing out this trace, it is found to be equal to a sum over the possible outcomes of the measurement, weighted by the Born probabilities for the system to be "thrown" into a particular state corresponding to each of these outcomes in the course of a measurement. This certainly represents our common-sense intuition about the meaning of expectation values as the sum over possible values that can appear in a given measurement, multiplied by the relative frequency of actual occurrence of these values in a series of such measurements. This interpretation, however, presumes (i) that measurements have outcomes, (ii) that measurements lead to definite "values," (iii) that measurable physical quantities are identified as operators (observables) in a Hilbert space, and (iv) that the modulus square of the expansion coefficients of the state in terms of the eigenbasis of the observable can be interpreted as representing probabilities of actual measurement outcomes (Born rule).

Thus decoherence brings about an apparent (and approximate) mixture of states that seem, based on the models studied, to correspond well to those states that we perceive as determinate. Moreover, our observation tells us that this apparent mixture indeed looks like a proper ensemble in a measurement situation, as we observe that measurements lead to the "realization" of precisely one state in the "ensemble." But within the framework of the orthodox interpretation, decoherence cannot explain this crucial step from an apparent mixture to the existence and/or perception of single outcomes.

2. Observables, measurements, and environment-induced superselection

In the standard and Copenhagen interpretationS, property ascription is determined by an observable that represents the measurement of a physical quantity and that in turn defines the preferred basis. However, any Hermitian operator can play the role of an observable, and thus any given state has the potential for an infinite number of different properties whose attribution is usually mutually exclusive unless the corresponding observables commute (in which case they share a common eigenbasis which preserves the uniqueness of the preferred basis). What then determines the observable that is being measured? As our discussion in Sec. II.C has demonstrated, the derivation of the measured observable from the particular form of a given state-vector expansion can lead to paradoxial results since this expansion is in general nonunique, so the observable must be chosen by other means. In the standard and Copenhagen interpretations, it is essentially the "user" who simply "chooses" the particular observable to be measured and thus determines which properties the system possesses.

This positivist point of view has, of course, led to a lot of controversy, since it runs counter to the attempt to establish an observer-independent reality that has been the central pursuit of natural science since its beginning. Moreover, in practice, one certainly does not have the freedom to choose any arbitrary observable and measure it; instead, one has "instruments" (including one's senses) that are designed to measure a particular observable. For most (and maybe all) practical purposes, this will ultimately boil down to a single relevant observable, namely, position. But what, then, makes the instruments designed for such a particular observable?

Answering this crucial question essentially means abandoning the orthodox view of treating measurements as a "black box" process that has little, if any, relation to the workings of actual physical measurements (where measurements can here be understood in the broadest sense of a "monitoring" of the state of the system). The first key point, the formalization of measurements as a formation of quantum correlations between system and apparatus, goes back to the early years of quantum mechanics and is reflected in the measurement scheme of von von Neumann (1932), but it does not resolve the issue of how the choice of observables is made. The second key point, the explicit inclusion of the environment in a description of the measurement process, was brought into quantum theory by the studies of decoherence. Zurek's (1981) stability criterion discussed in Sec. III.E has shown that measurements must be of such a nature as to establish stable records, where stability is to be understood as preserving the system-apparatus correlations in spite of the inevitable interaction with the surrounding environment. The "user" cannot choose the observables arbitrarily, but must design a measuring device whose interaction with the environment is such as to ensure stable records in the sense above (which, in turn, defines a measuring device for this observable). In the reading of orthodox quantum mechanics, this can be interpreted as the environment determining the properties of the system.

In this sense, the decoherence program has embedded the rather formal concept of measurement as proposed by the standard and Copenhagen interpretations—with its vague notion of observables that are seemingly freely chosen by the observer—in a more realistic and physical framework. This is accomplished via the specification of observer-free criteria for the selection of the measured observable through the physical structure of the measuring device and its interaction with the environment, which is, in most cases, needed to amplify the measurement record and thereby to make it accessible to the external observer.

3. The concept of classicality in the Copenhagen interpretation

The Copenhagen interpretation additionally postulates that classicality is not to be derived from quantum mechanics, for example, as the macroscopic limit of an underlying quantum structure (as is in some sense assumed, but not explicitely derived, in the standard interpretation), but instead that it be viewed as an indispensable and irreducible element of a complete quantum theory—and, in fact, be considered as a concept prior to quantum theory. In particular, the Copenhagen interpretation assumes the existence of macroscopic measurement apparatuses that obey classical physics and that are not supposed to be described in quantum mechanical terms (in sharp contrast to the von Neumann measurement scheme, which rather belongs to the standard interpretation); such a classical apparatus is considered necessary in order to make quantum-mechanical phenomena accessible to us in terms of the "classical" world of our experience. This strict dualism between the system $\mathcal{S}$, to be described by quantum mechanics, and the apparatus $\mathcal{A}$, obeying classical physics, also entails the existence of an essentially fixed boundary between $\mathcal{S}$ and $\mathcal{A}$, which separates the microworld from the macroworld (the "Heisenberg cut"). This boundary cannot be moved significantly without destroying the observed phenomenon (i.e., the full interacting compound $\mathcal{SA}$).

Especially in the light of the insights gained from decoherence it seems impossible to uphold the notion of a fixed quantum–classical boundary on a fundamental level of the theory. Environment-induced superselection and suppression of interference have demonstrated how quasiclassical robust states can emerge, or remain absent, using the quantum formalism alone and over a broad range of microscopic to macroscopic scales, and have established the notion that the boundary between $\mathcal{S}$ and $\mathcal{A}$ is to a large extent movable towards $\mathcal{A}$. Similar results have been obtained from the general study of quantum nondemolition measurements (see, for example, Chap. 19 of Auletta, 2000) which include the monitoring of a system by its environment. Also note that since the apparatus is described in classical terms, it is macroscopic by definition; but not every apparatus must be macrosopic: the actual "instrument" could well be microscopic. Only the "amplifier" must be macrosopic. As an example, consider Zurek's (1981) toy model of decoherence, outlined in Sec. III.D.2, in which the instrument can be represented by a bistable atom while the environment plays the role of the amplifier; a more realistic example is a macrosopic detector of gravitational waves that is treated as a quantum-mechanical harmonic oscillator.

Based on the progress already achieved by the decoherence program, it is reasonable to anticipate that decoherence embedded in some additional interpretive structure could lead to a complete and consistent derivation of the classical world from quantum mechanical principles. This would make the assumption of an intrinsically classical apparatus (which has to be treated outside of the realm of quantum mechanics) appear as neither a necessary nor a viable postulate. Bacciagaluppi (2003b, p. 22) refers to this strategy as "having Bohr's cake and eating it": acknowledging the correctness of Bohr's notion of the necessity of a classical world ("having Bohr's cake"), but being able to view the classical world as part of and as emerging from a purely quantum-mechanical world.

C. Relative-state interpretations

Everett's original (1957) proposal of a relative-state interpretation of quantum mechanics has motivated several strands of interpretation, presumably owing to the fact that Everett himself never clearly spelled out how his theory was supposed to work. The system-observer duality of orthodox quantum mechanics introduces into the theory external "observers" who are not described by the deterministic laws of quantum systems but instead follow a stochastic indeterminism. This approach obviously runs into problems when the universe as a whole is considered: by definition, there cannot be any external observers. The central idea of Everett's proposal is then to abandon duality and instead (i) to assume the existence of a total state $|\Psi\rangle$ representing the state of the entire universe and (ii) to uphold the universal validity of the Schrödinger evolution, while (iii) postulating that all terms in the superposition of the total state at the completion of the measurement actually correspond to physical states. Each such physical state can be understood as relative (a) to the state of the other part in the composite system (as in Everett's original proposal; also see Mermin, 1998a; Rovelli, 1996), (b) to a particular "branch" of a constantly "splitting" universe (the *many-worlds interpretations*, popularized by DeWitt, 1970 and Deutsch, 1985), or (c) to a particular "mind" in the set of minds of the conscious observer (the *many-minds interpretation*; see, for example, Lockwood, 1996). In other words, every term in the final-state superposition can be viewed as representing an equally "real" physical state of affairs that is realized in a different "branch of reality."

Decoherence adherents have typically been inclined towards relative-state interpretations (for instance Zeh, 1970, 1973, 1993; Zurek, 1998), presumably because the Everett approach takes unitary quantum mechanics essentially "as is," with a minimum of added interpretive elements. This matches well the spirit of the decoherence program, which attempts to explain the emergence of classicality purely from the formalism of basic quantum mechanics. It may also seem natural to identify the decohering components of the wave function with different Everett branches. Conversely, proponents of relative-state interpretations have frequently employed the mechanism of decoherence in solving the difficulties associated with this class of interpretations (see, for example, Deutsch, 1985, 1996, 2002; Saunders, 1995, 1997, 1998; Vaidman, 1998; Wallace, 2002, 2003a).

There are many different readings and versions of relative-state interpretations, especially with respect to what defines the "branches," "worlds," and "minds"; whether we deal with one, a multitude, or an infinity of such worlds and minds; and whether there is an actual (physical) or only perspectival splitting of the worlds and minds into different branches corresponding to the terms in the superposition. Does the world or mind split into two separate copies (thus somehow doubling all the matter contained in the orginal system), or is there just a "reassignment" of states to a multitude of worlds or minds of constant (typically infinite) number, or is there only one physically existing world or mind in which each branch corresponds to different "aspects" (whatever they are). Regardless, in the following discussion of the key implications of decoherence, the precise details and differences of these various strands of interpretation will, for the most part, be largely irrelevant.

Relative-state interpretations face two core difficulties. First, the preferred-basis problem: If states are only relative, the question arises, relative to what? What determines the particular basis terms that are used to define the branches, which in turn define the worlds or minds in the next instant of time? When precisely does the "splitting" occur? Which properties are made determinate in each branch, and how are they connected to the determinate properties of our experience? Second, what is the meaning of probabilities, since *every* outcome actually occurs in some world or mind, and how can Born's rule be motivated in such an interpretive framework?

1. Everett branches and the preferred-basis problem

Stapp (2002, p. 1043) stated the requirement that "a many-worlds interpretation of quantum theory exists only to the extent that the associated basis problem is solved." In the context of relative-state interpretations, the preferred-basis problem is not only much more severe than in the orthodox interpretation, but also more fundamental for several reasons: (i) The branching occurs continuously and essentially everywhere; in the general sense of measurements understood as the formation of quantum correlations, every newly formed correlation, whether it pertains to microscopic or macroscopic systems, corresponds to a branching. (ii) The ontological implications are much more drastic, at least in those relative-state interpretations, which assume an actual "splitting" of worlds or minds, since the choice of the basis determines the resulting "world" or "mind" as a whole.

The environment-based basis superselection criteria of the decoherence program have frequently been employed to solve the preferred-basis problem of relative-state interpretations (see, for example, Butterfield, 2001; Wallace, 2002, 2003a; Zurek, 1998). There are several advantages in a decoherence-related approach to selecting the preferred Everett bases: First, no *a priori* existence of a preferred basis needs to be postulated, but instead the preferred basis arises naturally from the physical criterion of robustness. Second, the selection will be likely to yield empirical adequacy, since the decoherence program is derived solely from the well-confirmed Schrödinger dynamics (modulo the possibility that robustness may not be the universally valid criterion). Lastly, the decohered components of the wave function evolve in such a way that they can be reidentified over time (forming "trajectories" in the preferred state spaces) and thus can be used to define stable, temporally extended Everett branches. Similarly, such trajectories can be used to ensure robust observer record states and/or environmental states that make information about the state of the system of interest widely accessible to observers (see, for example, Zurek's "existential interpretation," outlined in Sec. IV.C.3 below).

While the idea of directly associating the environment-selected basis states with Everett worlds seems natural and straightforward, it has also been subject to criticism. Stapp (2002) has argued that an Everett-type interpretation must aim at determining a denumerable set of distinct branches that correspond to the apparently discrete events of our experience. Among these branches one must be able to assign determinate values and finite probabilities according to the usual rules and therefore one would need to be able to specify a denumerable set of mutually orthogonal projection operators. It is well known, however (Zurek, 1998), that the preferred states chosen through the interaction with the environment via the stability criterion frequently form an overcomplete set of states—often a continuum of narrow Gaussian-type wave packets, for example, the coherent states of harmonic-oscillator models that are not necessarily orthogonal (i.e., the Gaussians may overlap; see Kübler and Zeh, 1973; Zurek *et al.*, 1993). Stapp therefore considers this approach to the preferred-basis problem in relative-state interpretations to be unsatisfactory. Zurek (2003a) has rebutted this criticism by pointing out that a collection of harmonic oscillators that would lead to such overcomplete sets of Gaussians cannot serve as an adequate model of the human brain, and it is ultimately only in the brain where the perception of denumerability and mutual exclusiveness of events must be accounted for (see Sec. II.B.3); when neurons are more appropriately modeled as two-state systems, the issue raised by Stapp disappears (for a discussion of decoherence in a simple two-state model, see Sec. III.D.2).[13]

[13] For interesting quantitative results on the role of decoherence in neuronal processes, see Tegmark (2000).

The approach of using environment-induced superselection and decoherence to define the Everett branches has also been critized on grounds of being "conceptually approximate," since the stability criterion generally leads only to an approximate specification of a preferred basis and therefore cannot give an "exact" definition of the Everett branches (see, for example, the comments of Kent, 1990; Zeh, 1973, and also the well-known "anti-FAPP" position of Bell, 1982). Wallace (2003a, pp. 90–91) has argued against such an objection as

> (...) arising from a view implicit in much discussion of Everett-style interpretations: that certain concepts and objects in quantum mechanics must either enter the theory formally in its axiomatic structure, or be regarded as illusion. (...) [Instead] the emergence of a classical world from quantum mechanics is to be understood in terms of the emergence from the theory of certain sorts of structures and patterns, and ... this means that we have no need (as well as no hope!) of the precision which Kent [in his (1990) critique] and others (...) demand.

Accordingly, in view of our argument in Sec. II.B.3 for considering subjective solutions to the measurement problem as sufficient, there is no *a priori* reason to doubt that an "approximate" criterion for the selection of the preferred basis can give a meaningful definition of the Everett branches—one that is empirically adequate and that accounts for our experiences. The environment-superselected basis emerges naturally from the physically very reasonable criterion of robustness together with the purely quantum mechanical effect of decoherence. It would be rather difficult to imagine rgar ab axiomatically introduced "exact" rule could be able to select preferred bases in a manner that is similarly physically motivated and capable of ensuring empirical adequacy.

Besides using the environment-superselected pointer states to describe the Everett branches, various authors have directly used the instantaneous Schmidt decomposition of the composite state (or, equivalently, the set of orthogonal eigenstates of the reduced density matrix) to define the preferred basis (see also Sec. III.E.4). This approach is easier to implement than the explicit search for dynamically stable pointer states since the preferred basis follows directly from a simple mathematical diagonalization procedure at each instant of time. Furthermore, it has been favored by some (e.g., Zeh, 1973) since it gives an "exact" rule for basis selection in relative-state interpretations; the consistently quantum origin of the Schmidt decomposition, which matches well the "pure quantum-mechanics" spirit of Everett's proposal (where the formalism of quantum mechanics supplies its own interpretation), has also been counted as an advantage (Barvinsky and Kamenshchik, 1995). In an earlier work, Deutsch (1985) attributed a fundamental role to the Schmidt decomposition in relative-state interpretations as defining an "interpretation basis" that imposes the precise structure that is needed to give meaning to Everett's basic concept.

However, as pointed out in Sec. III.E.4, emerging basis states based on the instantaneous Schmidt states will frequently have properties that are very different from those selected by the stability criterion and that are undesirably nonclassical. For example, they may lack the spatial localization of the robustness-selected Gaussians (Stapp, 2002). The question to what extent the Schmidt basis states correspond to classical properties in Everett-style interpretations was investigated in detail by Barvinsky and Kamenshchik (1995). The authors study the similarity of the states selected by the Schmidt decomposition to coherent states (i.e., minimum-uncertainty Gaussians) that are chosen as the "yardstick states" representing classicality (see also Eisert, 2004). For the investigated toy models it is found that only subsets of the Everett worlds corresponding to the Schmidt decomposition exhibit classicality in this sense; furthermore, the degree of robustness of classicality in these branches is very sensitive to the choice of the initial state and the interaction Hamiltonian, such that classicality emerges typically only temporarily, and the Schmidt basis generally lacks robustness under time evolution. Similar difficulties with the Schmidt basis approach have been described by Kent and McElwaine (1997).

These findings indicate that a selection criterion based on robustness provides a much more meaningful, physically transparent, and general rule for the selection of quasiclassical branches in relative-state interpretations, especially with respect to its ability to lead to wavefunction components representing quasiclassical properties that can be reidentified over time (which a simple diagonalization of the reduced density matrix at each instant of time does not, in general, allow for).

2. Probabilities in Everett interpretations

Various attempts unrelated to decoherence have been made to find a consistent derivation of the Born probabilities (for instance, Deutsch, 1999; DeWitt, 1971; Everett, 1957; Geroch, 1984; Graham, 1973; Hartle, 1968) in the explicit or implicit context of a relative-state interpretation, but several arguments have been presented that show that these approaches fail.[14] When the effects of decoherence and environment-induced superselection are included, it seems natural to identify the diagonal elements of the decohered reduced density matrix (in the environment-superselected basis) with the set of possible

[14] See, for example, the critiques of Barnum *et al.* (2000); Kent (1990); Squires (1990); Stein (1984); however, also note the arguments of Wallace (2003b) and Gill (2003), defending the approach of Deutsch (1999); see also Saunders (2002).

elementary events and to interpret the corresponding coefficients as relative frequencies of worlds (or minds, etc.) in the Everett theory, assuming a typically infinite multitude of worlds, minds, etc. Since decoherence enables one to reidentify the individual localized components of the wave function over time (describing, for example, observers and their measurement outcomes attached to individual well-defined "worlds"), this leads to a natural interpretation of the Born probabilities as empirical frequencies.

However, decoherence cannot yield an actual derivation of the Born rule (for attempts in this direction, see Deutsch, 1999; Zurek, 1998). As mentioned before, this is so because the key elements of the decoherence program, the formalism and the interpretation of reduced density matrices and the trace rule, *presume* the Born rule. Attempts to consistently derive probabilities from reduced density matrices and from the trace rule are therefore subject to the charge of circularity (Zeh, 1997; Zurek, 2003b). In Sec. III.F, we outlined a recent proposal by Zurek (2003c) that evades this circularity and deduces the Born rule from envariance, a symmetry property of entangled systems, and from certain assumptions about the connection between the state of the system $\mathcal{S}$ of interest, the state vector of the composite system $\mathcal{SE}$ that includes an environment $\mathcal{E}$ entangled with $\mathcal{S}$, and probabilities of outcomes of measurements performed on $\mathcal{S}$. Decoherence combined with this approach provides a framework in which quantum probabilities and the Born rule can be given a rather natural motivation, definition, and interpretation in the context of relative-state interpretations.

3. The "existential interpretation"

A well-known Everett-type interpretation that relies heavily on decoherence has been proposed by Zurek (1993, 1998; see also the recent reevaluation in Zurek, 2004a). This approach, termed the "existential interpretation," defines the reality, or objective existence, of a state as the possibility of finding out what the state is and simultaneously leaving it unperturbed, similar to a classical state.[15] Zurek assigns a "relative objective existence" to the robust states (identified with elementary "events") selected by the environmental stability criterion. By measuring properties of the system-environment interaction Hamiltonian and employing the robustness criterion, the observer can, at least in principle, determine the set of observables that can be measured on the system without perturbing it and thus find out its "objective" state. Alternatively, the observer can take advantage of the redundant records of the state of the system as monitored by the environment. By intercepting parts of this environment, for example, a fraction of the surrounding photons, he can determine the state of the system essentially without perturbing it (cf. also the related recent ideas of "quantum Darwinism" and the role of the environment as a "witness," see Ollivier *et al.*, 2003; Zurek, 2000, 2003b, 2004b).[16]

Zurek emphasizes the importance of stable records for observers, i.e., robust correlations between the environment-selected states and the memory states of the observer. Information must be represented physically, and thus the "objective" state of the observer who has detected one of the potential outcomes of a measurement must be physically distinct and objectively different from the state of an observer who has recorded an alternative outcome (since the record states can be determined from the outside without perturbing them—see the previous paragraph). The different objective states of the observer are, via quantum correlations, attached to different branches defined by the environment-selected robust states; they thus ultimately label the different branches of the universal state vector. This is claimed to lead to the perception of classicality; the impossibility of perceiving arbitrary superpositions is explained via the quick suppression of interference between different memory states induced by decoherence, where each (physically distinct) memory state represents an individual observer identity.

A similar argument has been given by Zeh (1993) who employs decoherence together with an (implicit) branching process to explain the perception of definite outcomes:

> [A]fter an observation one need not necessarily conclude that only one component now *exists* but only that only one component *is observed*. (...) Superposed world components describing the registration of different macroscopic properties by the "same" observer are dynamically entirely independent of one another: they describe different observers. (...) He who considers this conclusion of an indeterminism or splitting of the observer's identity, derived from the Schrödinger equation in the form of dynamically decoupling ("branching") wave packets on a fundamental global configuration space, as unacceptable or "extravagant" may instead dynamically formalize the superfluous hypothesis of a disappearance of the "other" components by whatever method he prefers, but he should be aware that he may thereby also create his own problems: Any deviation from the global Schrödinger equation must in

[15] This intrinsically requires the notion of open systems, since in isolated systems, the observer would need to know in advance what observables commute with the state of the system, in order to perform a nondemolition measurement that avoids repreparing the state of the system.

[16] The partial ignorance is necessary to avoid redefinition of the state of the system.

> principle lead to observable effects, and it should be recalled that none have ever been discovered.

The existential interpretation has recently been connected to the theory of envariance (see Zurek, 2004a, and Sec. III.F). In particular, the derivation of Born's rule based on envariance as outlined in Sec. III.F can be recast in the framework of the existential interpretation such that probabilities refer explicitly to the future record state of an observer. Such a concept of probability bears similarities with classical probability theory (for more details on these ideas, see Zurek, 2004a).

The existential interpretation continues Everett's goal of interpreting quantum mechanics using the quantum-mechanical formalism itself. Zurek takes the standard no-collapse quantum theory "as is" and explores to what extent the incorporation of environment-induced superselection and decoherence (and recently also envariance) could form a viable interpretation that would, with a minimal additional interpretive framework, be capable of accounting for the perception of definite outcomes and of explaining the origin and nature of probabilities.

D. Modal interpretations

The first type of modal interpretation was suggested by van Fraassen (1973, 1991), based on his program of "constructive empiricism," which proposes to take only empirical adequacy, but not necessarily "truth," as the goal of science. Since then, a large number of interpretations of quantum mechanics have been suggested that can be considered as modal (for a review and discussion of some of the basic properties and problems of such interpretations, see Clifton, 1996).

In general, the approach of modal interpretations consists in weakening the orthodox e-e link by allowing for the assignment of definite measurement outcomes even if the system is not in an eigenstate of the observable representing the measurement. In this way, one can preserve a purely unitary time evolution without the need for an additional collapse postulate to account for definite measurement results. Of course, this immediately raises the question of how physical properties that are perceived through measurements and measurement results are connected to the state, since the bidirectional link is broken between the eigenstate of the observable (which corresponds to the physical property) and the eigenvalue (which represents the manifestation of the value of this physical property in a measurement). The general goal of modal interpretations is then to specify rules that determine a catalog of possible properties of a system described by the density matrix ρ at time t. Two different views are typically distinguished: a semantic approach that only changes the way of talking about the connection between properties and state; and a realistic view that provides a different specification of what the possible properties of a system really are, given the state vector (or the density matrix).

Such an attribution of possible properties must fulfill certain requirements. For instance, probabilities for outcomes of measurements should be consistent with the usual Born probabilities of standard quantum mechanics; it should be possible to recover our experience of classicality in the perception of macroscopic objects; and an explicit time evolution of properties and their probabilities should be definable that is consistent with the results of the Schrödinger equation. As we shall see in the following, decoherence has frequently been employed in modal interpretations to motivate and define rules for property ascription. Dieks (1994a,b) has argued that one of the central goals of modal approaches is to provide an interpretation for decoherence.

1. Property assignment based on environment-induced superselection

The intrinsic difficulty of modal interpretations is to avoid any *ad hoc* character of the property assignment, yet to find generally applicable rules that lead to a selection of possible properties that include the determinate properties of our experience. To solve this problem, various modal interpretations have embraced the results of the decoherence program. A natural approach would be to employ the environment-induced superselection of a preferred basis—since it is based on an entirely physical and very general criterion (namely, the stability requirement) and has, for the cases studied, been shown to give results that agree well with our experience, thus matching van Fraassen's goal of empirical adequacy—to yield sets of possible quasiclassical properties associated with the correct probabilities.

Furthermore, since the decoherence program is based solely on Schrödinger dynamics, the task of defining a time evolution of the "property states" and their associated probabilities that is in agreement with the results of unitary quantum mechanics would presumably be easier than in a model of property assignment in which the set of possibilities does not arise dynamically via the Schrödinger equation alone (for a detailed proposal for modal dynamics of the latter type, see Bacciagaluppi and Dickson, 1999). The need for explicit dynamics of property states in modal interpretations is controversial. One can argue that it suffices to show that at each instant of time, the set of possibly possessed properties that can be ascribed to the system is empirically adequate, in the sense of containing the properties of our experience, especially with respect to the properties of macroscopic objects (this is essentially the view of, for example, van Fraassen, 1973, 1991). On the other hand, this cannot ensure that these properties behave over time in agreement with our experience (for instance, that macroscopic objects that are left undisturbed do not change their position in space spontaneously in an observable manner). In other words, the emergence of classicality is to be tied not only to determinate properties at each instant of time, but also to the existence of quasiclassical "trajectories" in property space. Since decoherence allows one to reidentify components of the decohered density matrix over time, this could be used to derive property states with continuous, quasiclassical trajectorylike time evolution based on Schrödinger dynamics alone. For some discussions of this approach, see Hemmo (1996) and Bacciagaluppi and Dickson (1999).

The fact that the states emerging from decoherence and the stability criterion are sometimes nonorthogonal or form a continuum will presumably be of even less relevance in modal interpretations than in Everett-style interpretations (see Sec. IV.C) since the goal here is solely to specify sets of possible properties, of which only one set actually gets assigned to the system. Hence an "overlap" of the sets is not necessarily a problem (modulo the potential difficulty of a straightforward assignment of probabilities in such a situation).

2. Property assignment based on instantaneous Schmidt decompositions

Since it is usually rather difficult to determine explicitly the robust "pointer states" through the stability (or a similar) criterion, it would not be easy to specify a general rule for property assignment based on environment-induced superselection. To simplify this situation, several modal interpretations have restricted themselves to the orthogonal decomposition of the density matrix to define the set of properties that can be assigned (see, for instance, Bub, 1997; Dieks, 1989; Healey, 1989; Kochen, 1985; Vermaas and Dieks, 1995). For example, the approach of Dieks (1989) recognizes, by referring to the decoherence program, the relevance of the environment by considering a composite system-environment state vector and its diagonal Schmidt decomposition, $|\psi\rangle = \sum_k \sqrt{p_k}\, |\phi_k^{\mathcal{S}}\rangle|\phi_k^{\mathcal{E}}\rangle$, which always exists. Possible properties that can be assigned to the system are then represented by the Schmidt projectors $\widehat{P}_k = \lambda_k |\phi_k^{\mathcal{S}}\rangle\langle\phi_k^{\mathcal{S}}|$. Although all terms are present in the Schmidt expansion (that Dieks calls the "mathematical state"), the "physical state" is postulated to be given by only one of the terms, with probability p_k. A generalization of this approach to a decomposition into any number of subsystems has been described by Vermaas and Dieks (1995). In this sense, the Schmidt decomposition itself is taken to define an interpretation of quantum mechanics. Dieks (1995) suggested a physical motivation for the Schmidt decomposition in modal interpretations based on the assumed requirement of a one-to-one correspondence between the properties of the system and its environment. For a comment on the violation of the property composition principle in such interpretations, see the analysis of Clifton (1996).

A central problem associated with the approach of orthogonal decomposition is that it is not at all clear that the properties determined by the Schmidt diagonalization represent the determinate properties of our experience. As outlined in Sec. III.E.4, the states selected by the (instantaneous) orthogonal decomposition of the reduced density matrix will in general differ from the robust "pointer states" chosen by the stability criterion of the decoherence program and may have distinctly nonclassical properties. That this will be the case especially when the states selected by the orthogonal decomposition are close to degeneracy has already been indicated in Sec. III.E.4. It has also been explored in more detail in the context of modal interpretations by Bacciagaluppi *et al.* (1995) and Donald (1998), who showed that in the case of near degeneracy (as it typically occurs for macroscopic systems with many degrees of freedom), the resulting projectors will be extremely sensitive to the precise form of the state (Bacciagaluppi *et al.*, 1995). Clearly such sensitivity is undesired since the projectors, and thus the properties of the system, will not be well behaved under the inevitable approximations employed in physics (Donald, 1998).

3. Property assignment based on decompositions of the decohered density matrix

Other authors therefore have appealed to the orthogonal decomposition of the decohered reduced density matrix (instead of the decomposition of the instantaneous density matrix) which has led to noteworthy results. When the system is represented by only a finite-dimensional Hilbert space, a discrete model of decoherence, the resulting states were indeed found to be typically close to the robust states selected by the stability criterion (for macroscopic systems, this typically meant localization in position space), unless again the final composite state was very nearly degenerate (Bacciagaluppi and Hemmo, 1996; Bene, 2001; see also Sec. III.E.4). Thus, in sufficiently nondegenerate cases, decoherence can ensure that the definite properties selected by modal interpretations of the Dieks type will be appropriately close to the properties corresponding to the ideal pointer states if the modal properties are based on the orthogonal decomposition of the reduced decohered density matrix.

On the other hand, Bacciagaluppi (2000) showed that in the more general and realistic case of an infinite-dimensional state space of the system, when one employs a continuous model of decoherence (namely, that of Joos and Zeh, 1985), the predictions of the modal approach (Dieks, 1989; Vermaas and Dieks, 1995) and those of decoherence can differ significantly. It was demonstrated that the definite properties obtained from the orthogonal decomposition of the decohered density matrix were highly delocalized (that is, smeared out over the entire spread of the state), although the coherence length of the density matrix itself was shown to be very small, so that decoherence indicated very localized properties. Thus, based on these results (and similar ones of Donald, 1998), decoherence can be used to argue for the physical inadequacy of the rule for the assignment of definite properties as proposed by Dieks (1989) and Vermaas and Dieks (1995).

More generally, if the definite properties selected by the modal interpretation fail to mesh with the results of decoherence (in particular, when they also lack the desired classicality and correspondence to the determinate properties of our experience), we are given reason to doubt whether the proposed rules for property assignment have sufficient physical motivation, legitimacy, or generality.

4. Concluding remarks

There are many different proposals that can be grouped under the heading of modal interpretations. They all share the problem of motivating and verifying a consistent system of property assignment. Using the robust pointer states selected by interaction with the environment and by the stability criterion is a step in the right direction, but the difficulty remains to derive a general rule for property assignment from this method that would yield explicitly the sets of possibilities in every situation. In certain cases, for example, close to degeneracy and in Hilbert-state spaces of infinite dimension, the simpler approach of deriving the possible properties from the orthogonal decomposition of the decohered reduced density matrix fails to yield the sharply localized, quasiclassical pointer states as selected by environmental robustness criteria. These are the cases in which decoherence can play a vital role in helping to identify inadequate rules for property assignment in modal interpretations.

E. Physical collapse theories

The basic idea of physical collapse theories is to introduce an explicit modification of the Schrödinger time evolution to achieve a physical mechanism for state-vector reduction (for an extensive recent review, see Bassi and Ghirardi, 2003). This is in general motivated by a "realist" interpretation of the state vector, that is, the state vector is directly identified with a physical state, which then requires reduction to one of the terms in the superposition to establish equivalence to the observed determinate properties of physical states, at least as far as the macroscopic realm is concerned.

The first proposals for theories of this type were made by Pearle (1976, 1982, 1979) and Gisin (1984), who developed dynamical reduction models that modify unitary dynamics such that a superposition of quantum states evolves continuously into one of its terms (see also the review by Pearle, 1999). Typically, terms representing external white noise are added to the Schrödinger equation, causing the squared amplitudes $|c_n(t)|^2$ in the state-vector expansion $|\Psi(t)\rangle = \sum_n c_n(t)|\psi_n\rangle$ to fluctuate randomly in time, while maintaining the normalization condition $\sum_n |c_n(t)|^2 = 1$ for all t. This process is known as *stochastic dynamical reduction.* Eventually one amplitude $|c_n(t)|^2 \to 1$, while all other squared coefficients $\to 0$ (the "gambler's ruin game" mechanism), where $|c_n(t)|^2 \to 1$ with probability $|c_n(t=0)|^2$ (the squared coefficients in the initial precollapse state-vector expansion) in agreement with the Born probability interpretation of the expansion coefficients.

These early models exhibit two main difficulties. First, the preferred-basis problem: What determines the terms in the state-vector expansion into which the state vector gets reduced? Why does reduction lead to precisely the distinct macroscopic states of our experience and not superpositions thereof? Second, how can one account for the fact that the effectiveness of collapsing superpositions increases when going from microscopic to macroscopic scales?

These problems motivated *spontaneous localization* models, initially proposed by Ghirardi, Rimini, and Weber (GRW; Ghirardi *et al.*, 1986). Here state-vector reduction is not implemented as a dynamical process (i.e., as a continuous evolution over time), but instead occurs instantaneously and spontaneously, leading to a spatial localization of the wave function. To be precise, the N-particle wave function $\psi(\mathbf{x}_1, \dots, \mathbf{x}_N)$ is at random intervals multiplied by a Gaussian of the form $\exp\left[-(\mathbf{X}-\mathbf{x}_k)^2/2\Delta^2\right]$ (this process is often called a "hit" or a "jump"), and the resulting product is subsequently normalized. The occurrence of these hits is not explained, but rather postulated as a new fundamental physical mechanism. Both the coordinate $\mathbf{x}_k$ and the "center of the hit" $\mathbf{X}$ are chosen at random, but the probability for a specific $\mathbf{X}$ is postulated to be given by the squared inner product of $\psi(\mathbf{x}_1, \dots, \mathbf{x}_N)$ with the Gaussian (and therefore hits are more likely to occur where $|\psi|^2$, viewed as a function of $\mathbf{x}_k$ only, is large). The mean frequency ν of hits for a single microscopic particle is chosen so as to effectively preserve unitary time evolution for microscopic systems, while ensuring that for macroscopic objects containing a very large number N of particles the localization occurs rapidly (on the order of $N\nu$), in such a way as to preclude the persistence of spatially separated macroscopic superpositons (such as the pointer's being in a superpositon of "up" and "down") on time scales shorter than realistic observations could resolve. Ghirardi *et al.* (1986) chose $\nu \approx 10^{-16}\ \mathrm{s}^{-1}$, so a macrosopic system with $N \approx 10^{23}$ particles undergoes localization on average every 10^{-7} s. Inevitable coupling to the environment can in general be expected to lead to a further drastic increase of N and therefore to an even higher localization rate. Note, however, that the localization process itself is independent of any interaction with environment, in sharp contrast to the decoherence approach.

Subsequently, the ideas of the stochastic dynamical reduction and GRW theory were combined into *continuous spontaneous localization* models (Ghirardi *et al.*, 1990; Pearle, 1989) in which localization of the GRW type can be shown to emerge from a nonunitary, nonlinear Itô stochastic differential equation, namely, the Schrödinger equation augmented by spatially correlated Brownian motion terms (see also Diósi, 1988, 1989). The particular choice of stochastic term determines the preferred basis. Frequently, the stochastic term has been based on the mass density which yields a GRW-type spatial localization (Diósi, 1989; Ghirardi *et al.*, 1990; Pearle, 1989), but stochastic terms driven by the Hamiltonian, leading to a reduction on an energy basis, have also been studied (Adler, 2002; Adler *et al.*, 2001; Adler and Horwitz, 2000; Bedford and Wang, 1975, 1977; Fivel, 1997; Hughston, 1996; Milburn, 1991; Percival, 1995, 1998). If we focus on the first type of term, the Ghirardi-Rimini-Weber theory and continuous spontaneous localization become phenomenologically similar, and we shall refer to them jointly as "spontaneous localization" models in the following discussion whenever it is unnecessary to distinguish them explicitly.

1. The preferred-basis problem

Physical reduction theories typically remove wave-function collapse from the restrictive context of the orthodox interpretation (where the external observer arbitrarily selects the measured observable and thus determines the preferred basis), and rather understand reduction as a universal mechanism that acts constantly on every state vector regardless of an explicit measurement situation. In view of this it is particularly important to provide a definition for the states into which the wave function collapses.

As mentioned before, the original stochastic dynamical reduction models suffer from this preferred-basis problem. Taking into account environment-induced superselection of a preferred basis could help resolve this issue. Decoherence has been shown to occur, especially for mesoscopic and macroscopic objects, on extremely short time scales, and thus would presumably be able to bring about basis selection much faster than the time required for dynamical fluctuations to establish a "winning" expansion coefficient.

In contrast, the GRW theory solves the preferred-basis problem by postulating a mechanism that leads to reduction to a particular state vector in an expansion on a position basis, i.e., position is assumed to be the universal preferred basis. State-vector reduction then amounts to simply modifying the functional shape of the projection of the state vector $|\psi\rangle$ onto the position basis $\langle \mathbf{x}_1, \ldots, \mathbf{x}_N|$. This choice can be motivated by the insight that essentially all (human) observations must be grounded in a position measurement.[17]

On the one hand, the selection of position as the preferred basis is supported by the decoherence program, since physical interactions frequently are governed by distance-dependent laws. Given the stability criterion or a similar requirement, this leads to position as the preferred observable. In this sense, decoherence provides a physical motivation for the assumption of the GRW model. On the other hand, it makes this assumption appear as too restrictive as it cannot account for cases in which position is not the preferred basis—for instance, in microscopic systems where typically energy is the robust observable, or in the superposition of (macroscopic) currents in SQUIDs. The GRW model simply excludes such cases by choosing the parameters of the spontaneous localization process such that microscopic systems remain generally unaffected by any state vector reduction. The basis selection approach proposed by the decoherence program is therefore much more general and also avoids the *ad hoc* character of the GRW theory by allowing for a range of preferred observables and motivating their choice on physical grounds.

A similar argument can be made with respect to the continuous spontaneous localization approach. Here, one essentially preselects a preferred basis through the particular choice of the stochastic terms added to the Schrödinger equation. This allows for a greater range of possible preferred bases, for instance by combining terms driven by the Hamiltonian and by the mass density, leading to a competition between localization in energy and position space (corresponding to the two most frequently observed eigenstates). Nonetheless, any particular choice of terms will again be subject to the charge of possessing an *ad hoc* flavor, in contrast to the physical definition of the preferred basis derived from the structure of the unmodified Hamiltonian as suggested by environment-induced selection.

2. Simultaneous presence of decoherence and spontaneous localization

Since decoherence can be considered as an omnipresent phenomenon that has been extensively verified both theoretically and experimentally, the assumption that a physical collapse theory holds means that the evolution of a system must be guided by both decoherence effects *and* the reduction mechanism.

Let us first consider the situation in which decoherence and the localization mechanism act constructively in the same direction, i.e., towards a common preferred basis. This raises the question in which order these two effects influence the evolution of the system (Bacciagaluppi, 2003a). If localization occurs on a shorter time scale than environment-induced superselection of a preferred basis

[17] This measurement may ultimately occur only in the brain of the observer; see the objection to the GRW model by Albert and Vaidman (1989). With respect to the general preference for position as the basis of measurements, see also the comment by Bell (1982).

and suppression of local interference, decoherence will in most cases have very little influence on the evolution of the system, since typically the system will already have evolved into a reduced state. Conversely, if decoherence effects act more quickly on the system than the localization mechanism, the interaction with the environment will presumably lead to the preparation of quasiclassical robust states that are subsequently chosen by the localization mechanism. As pointed out in Sec. III.D, decoherence usually occurs on extremely short time scales, which can be shown to be significantly smaller than the action of the spontaneous localization process for most cases (for studies related to the GRW model, see Tegmark, 1993 and Benatti *et al.*, 1995). This indicates that decoherence will typically play an important role even in the presence of physical wave-function reduction.

The second case occurs when decoherence leads to the selection of a different preferred basis than the reduction basis specified by the localization mechanism. As remarked by Bacciagaluppi (2003a,b) in the context of the GRW theory, one might then imagine the collapse either to occur only at the level of the environment (which would then serve as an amplifying and recording device with different localization properties than the system under study), or to lead to an explicit competition between decoherence and localization effects.

3. The tails problem

The clear advantage of physical collapse models over the consideration of decoherence-induced effects alone for a solution to the measurement problem lies in the fact that an actual state reduction is achieved such that one may be tempted to conclude that at the conclusion of the reduction process the system actually is in a determinate state. However, all collapse models achieve only an approximate ("for all practical purposes") reduction of the wave function. In the case of dynamical reduction models, the state will always retain small interference terms for finite times. Similarly, in the GRW theory the width Δ of the multiplying Gaussian cannot be made arbitrarily small, and therefore the reduced wave packet cannot be made infinitely sharply localized in position space, since this would entail an infinitely large energy gain by the system via the time-energy uncertainty relation, which would certainly show up experimentally (Ghirardi *et al.*, 1986, chose $\Delta \approx 10^{-5}$ cm). This need for only an approximate reduction leads to wave function "tails" (Albert and Loewer, 1996), that is, in any region in space and at any time $t > 0$, the wave function will remain nonzero if it has been nonzero at $t = 0$ (before the collapse), and thus there will be always a part of the system that is not "here."

Physical collapse models that achieve reduction only "for all practical purposes" require a modification, namely, a weakening, of the orthodox e-e link to allow one to speak of the system's actually being in a definite state, and thereby to ensure the objective attribution of determinate properties to the system.[18] In this sense, collapse models are as much "fine for all practical purposes" (to paraphrase Bell, 1990) as decoherence is, where perfect orthogonality of the environment states is only attained as $t \to \infty$. The severity of the consequences, however, is not equivalent for the two strategies. Since collapse models directly change the state vector, a single outcome is at least approximately selected, and it only requires a "slight" weakening of the e-e link to make this state of affairs correspond to the (objective) existence of a determinate physical property. In the case of decoherence, the lack of a precise destruction of interference terms is not the main problem; even if exact orthogonality of the environment states were ensured at all times, the resulting reduced density matrix would represent an improper mixture, with no outcome having been singled out according to the e-e link. This would be the case regardless of whether the e-e link is expressed in the strong or weakened form, and we would still have to supply some additional interpretative framework to explain our perception of outcomes (see also the comment by Ghirardi *et al.*, 1987).

4. Connecting decoherence and collapse models

It was realized early that there exists a striking formal similarity of the equations that govern the time evolution of density matrices in the GRW approach and in models of decoherence. For example, the GRW equation for a single free mass point reads [Ghirardi *et al.*, 1986, Eq. (3.5)]

$$i\frac{\partial \rho(x,x',t)}{\partial t} = \frac{1}{2m}\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2}\right]\rho - i\Lambda(x-x')^2\rho, \quad (4.1)$$

where the second term on the right-hand side accounts for the destruction of spatially separated interference terms. A simple model for environment-induced decoherence yields a very similar equation [Joos and Zeh, 1985, Eq. (3.75); see also the comment by Joos, 1987]. Thus the physical justification for an *ad hoc* postulate of an explicit reduction-inducing mechanism could be questioned (of course modulo the important interpretive difference between the approximately proper ensembles arising from collapse models and the improper ensembles resulting from decoherence; see also Ghirardi *et al.*, 1987). More constructively, the similarity of the governing equations might enable one to choose the free parameters in collapse models on physical grounds rather than on the basis of empirical adequacy. Conversely, this similiarity can also

[18] It should be noted, however, that such "fuzzy" e-e links may in turn lead to difficulties, as the discussion of Lewis's "counting anomaly" has shown (Lewis, 1997).

be viewed as leading to a "protection" of physical collapse theories from empirical disproof. This is so because the inevitable and ubiquitous interaction with the environment will always, for all practical purposes of observation (that is, of statistical prediction), result in (local) density matrices that are formally very similar to those of collapse models. What is measured is not the state vector itself, but the probability distribution of outcomes, i.e., values of a physical quantity and their frequency, and this information is equivalently contained in the state vector and the density matrix. Measurements with their intrinsically local character will presumably be unable to distinguish between the probability distribution given by the decohered reduced density matrix and the probability distribution defined by an (approximately) proper mixture obtained from a physical collapse. In other words, as long as the free parameters of collapse theories are chosen in agreement with those determined from decoherence, models for state-vector reduction can be expected to be empirically adequate since decoherence is an effect that will be present with near certainty in every realistic (especially macroscopic) physical system.

One might of course speculate that the simultaneous presence of both decoherence and reduction effects might actually allow for an experimental disproof of collapse theories by preparing states that differ in an observable manner from the predictions of the reduction models.[19] If we acknowledge the existence of interpretations of quantum mechanics that employ only decoherence-induced suppression of interference to explain the perception of apparent collapses (as is, for example, claimed by the "existential interpretation" of Zurek, 1993, 1998; see Sec. IV.C.3), we will not be able to distinguish experimentally between a "true" collapse and a mere suppression of interference as explained by decoherence. Instead, an experimental situation is required in which the collapse model predicts a collapse, but in which no suppression of interference through decoherence arises. Again, the problem in the realization of such an experiment is that it is very difficult to shield a system from decoherence effects, especially since we will typically require a mesoscopic or macroscopic system in which the reduction is efficient enough to be observed. For example, based on explicit numerical estimates, Tegmark (1993) has shown that decoherence due to scattering of environmental particles such as air molecules or photons will have a much stronger influence than the proposed GRW effect of spontaneous localization (see also Bassi and Ghirardi, 2003; Benatti *et al.*, 1995; for different results for energy-driven reduction models, cf. Adler, 2002).

[19] For proposed experiments to detect the GRW collapse, see for example Squires (1991) and Rae (1990). For experiments that could potentially demonstrate deviations from the predictions of quantum theory when dynamical state-vector reduction is present, see Pearle (1984, 1986).

5. Summary and outlook

Decoherence has the distinct advantage of being derived directly from the laws of standard quantum mechanics, whereas current collapse models are required to postulate their reduction mechanism as a new fundamental law of nature. On the other hand, collapse models yield, at least for all practical purposes, proper mixtures, so they are capable of providing an "objective" solution to the measurement problem. The formal similarity between the time evolution equations of the collapse and decoherence models nourishes hopes that the postulated reduction mechanisms of collapse models could possibly be derived from the ubiquitous and inevitable interaction of every physical system with its environment and the resulting decoherence effects. We may therefore regard collapse models and decoherence not as mutually exclusive alternatives for a solution to the measurement problem, but rather as potential candidates for a fruitful unification. For a vague proposal along these lines, see Pessoa (1998); cf. also Diósi (1989) and Pearle (1999) for speculations that quantum gravity might act as a collapse-inducing universal "environment."

F. Bohmian mechanics

Bohm's approach (Bohm, 1952; Bohm and Bub, 1966; Bohm and Hiley, 1993) is a modification of de Broglie's (1930) original "pilot-wave" proposal. In Bohmian mechanics, a system containing N (nonrelativistic) particles is described by a wave function $\psi(t)$ and the configuration $\mathcal{Q}(t) = \big(\mathbf{q}_1(t), \dots, \mathbf{q}_N(t)\big) \in \mathbb{R}^{3N}$ of particle positions $\mathbf{q}_i(t)$, i.e., the state of the system is represented by $(\psi, \mathcal{Q})$ for each instant t. The evolution of the system is guided by two equations. The wave function $\psi(t)$ is transformed as usual via the standard Schrödinger equation, $i\hbar(\partial/\partial t)\psi = \widehat{H}\psi$, while the particle positions $\mathbf{q}_i(t)$ of the configuration $\mathcal{Q}(t)$ evolve according to the "guiding equation"

$$\frac{d\mathbf{q}_i}{dt} = \mathbf{v}_i^{\psi}(\mathbf{q}_1, \dots, \mathbf{q}_N) \equiv \frac{\hbar}{m_i} \operatorname{Im} \frac{\psi^* \nabla_{\mathbf{q}_i} \psi}{\psi^* \psi}(\mathbf{q}_1, \dots, \mathbf{q}_N), \tag{4.2}$$

where m_i is the mass of the ith particle. Thus the particles follow determinate trajectories described by $\mathcal{Q}(t)$, with the distribution of $\mathcal{Q}(t)$ being given by the quantum equilibrium distribution $\rho = |\psi|^2$.

1. Particles as fundamental entities

Bohm's theory has been critized for ascribing fundamental ontological status to particles. General arguments against particles on a fundamental level of any relativistic quantum theory have been frequently given (see, for instance, Malament, 1996, and Halvorson and Clifton, 2002).[20] Moreover, and this is the point we would like to discuss in this section, it has been argued that the appearance of particles ("discontinuities in space") could be derived from the continuous process of decoherence, leading to claims that no fundamental role need be attributed to particles (Zeh, 1993, 1999, 2003). Based on decohered density matrices of mesoscopic and macroscopic systems that essentially always represent quasi-ensembles of narrow wave packets in position space, Zeh (1993, p. 190) holds that such wave packets can be viewed as representing individual "particle" positions:[21]

> All particle aspects observed in measurements of quantum fields (like spots on a plate, tracks in a bubble chamber, or clicks of a counter) can be understood by taking into account this decoherence of the relevant local (*i.e.*, subsystem) density matrix.

The first question is then whether a narrow wave packet in position space can be identified with the subjective experience of a "particle." The answer appears to be yes: our notion of "particles" hinges on the property of localizability, i.e., the definition of a region of space $\mathbf{\Omega} \in \mathbb{R}^3$ in which the system (that is, the support of the wave function) is entirely contained. Although the nature of the Schrödinger dynamics implies that any wave function will have nonvanishing support ("tails") outside of any finite spatial region $\mathbf{\Omega}$ and therefore exact localizatibility will never be achieved, we only need to demand approximate localizability to account for our experience of particle aspects.

However, to interpret the ensembles of narrow wave packets resulting from decoherence as leading to the perception of individual particles, we must embed standard quantum mechanics (with decoherence) into an additional interpretive framework that explains why only one of the wavepackets is perceived;[22] that is, we do need to add some interpretive rule to get from the improper ensemble emerging from decoherence to the perception of individual terms, so decoherence alone does not necessarily make Bohm's particle concept superfluous. But it suggests that the postulate of particles as fundamental entities could be unnecessary, and taken together with the difficulties in reconciling such a particle theory with a relativistic quantum field theory, Bohm's *a priori* assumption of particles at a fundamental level of the theory appears seriously challenged.

2. Bohmian trajectories and decoherence

A well-known property of Bohmian mechanics is the fact that its trajectories are often highly nonclassical (see, for example, Appleby, 1999b; Bohm and Hiley, 1993; Holland, 1993). This poses the serious problem of how Bohm's theory can explain the existence of quasiclassical trajectories on a macroscopic level.

Bohm and Hiley (1993) considered the scattering of a beam of environmental particles on a macroscopic system, a process that is known to give rise to decoherence (Joos and Zeh, 1985; Joos *et al.*, 2003). The authors demonstrate that this scattering yields quasiclassical trajectories for the system. It has further been shown that for isolated systems, the Bohm theory will typically not give the correct classical limit (Appleby, 1999b). It was thus suggested that the inclusion of the environment and of the resulting decoherence effects might be helpful in recovering quasiclassical trajectories in Bohmian mechanics (Allori, 2001; Allori *et al.*, 2002; Allori and Zanghì, 2001; Appleby, 1999a; Sanz and Borondo, 2003; Zeh, 1999).

We mentioned before that the interaction between a macroscopic system and its environment will typically lead to a rapid approximate diagonalization of the reduced density matrix in position space, and thus to spatially localized wave packets that follow (approximately) Hamiltonian trajectories. [This observation also provides a physical motivation for the choice of position as the fundamental preferred basis in Bohm's theory, in agreement with Bell's (1982) well-known comment that "in physics the only observations we must consider are position observations, if only the positions of instrument pointers."] The intuitive step is then to associate these trajectories with the particle trajectories $\mathcal{Q}(t)$ of the Bohm theory. As pointed out by Bacciagaluppi (2003b), a great advantage of this strategy lies in the fact that the same approach would allow for a recovery of both quantum and classical phenomena.

However, a careful analysis by Appleby (1999a) showed that this decoherence-induced diagonalization in the position basis alone will in general not suffice to yield quasiclassical trajectories in Bohm's theory; only under certain additional assumptions will processes that lead to decoherence also give correct quasiclassical Bohmian trajectories for macroscopic systems (Appleby described the example of the long-time limit of a system that has initially been prepared in an energy eigenstate). Interesting results were also reported by Allori and co-workers (Allori, 2001; Allori *et al.*, 2002; Allori and Zanghì, 2001). They demonstrated that decoherence effects can play the role of preserving classical properties of Bohmian trajectories. Furthermore, they showed that while in standard quantum mechanics it is important to maintain narrow

[20] On the other hand, there are proposals for a "Bohmian mechanics of quantum fields," i.e., a theory that embeds quantum field theory into a Bohmian-style framework (Dürr *et al.*, 2003, 2004).

[21] Schrödinger (1926) had made an attempt into a similar direction but had failed since the Schrödinger equation tends to continuously spread out any localized wavepacket when it is considered as describing an isolated system. The inclusion of an interacting environment and thus decoherence counteracts the spread and opens up the possibility of maintaining narrow wave packets over time (Joos and Zeh, 1985).

[22] Zeh himself, like Zurek (1998), adheres to an Everett-style branching to which distinct observers are attached (Zeh, 1993); see also the quote in Sec. IV.C.

wave packets to account for the emergence of classicality, the Bohmian description of a system by both its wave function and its configuration allows for the derivation of quasiclassical behavior from highly delocalized wave functions. Sanz and Borondo (2003) studied the double-slit experiment in the framework of Bohmian mechanics and in the presence of decoherence and showed that even when coherence is fully lost, and thus interference is absent, nonlocal quantum correlations remain that influence the dynamics of the particles in the Bohm theory, demonstrating that in this example decoherence does not suffice to achieve the classical limit in Bohmian mechanics.

In conclusion, while the basic idea of employing decoherence-related processes to yield the correct classical limit of Bohmian trajectories seems reasonable, many details of this approach still need to be worked out.

G. Consistent histories interpretations

The consistent- (or decoherent-) histories approach was introduced by Griffiths (1984, 1993, 1996) and further developed by Omnès (1988a,b,c, 1990, 1992, 1994, 2002), Gell-Mann and Hartle (1990, 1991a, 1993, 1991b), Dowker and Halliwell (1992), and others. Reviews of the program can be found in the papers by Omnès (1992) and Halliwell (1993, 1996), as well as in the recent book by Griffiths (2002). Thoughtful critiques investigating key features and assumptions of the approach have been given, for example, by d'Espagnat (1989), Dowker and Kent (1995, 1996), Kent (1998), and Bassi and Ghirardi (1999). The basic idea of the consistent-histories approach is to eliminate the fundamental role of measurements in quantum mechanics, and instead study quantum histories, defined as sequences of events represented by sets of time-ordered projection operators, and to assign probabilities to such histories. The approach was originally motivated by quantum cosmology, i.e., the study of the evolution of the entire universe, which, by definition, represents a closed system. Therefore no external observer (which is, for example, an indispensable element of the Copenhagen interpretation) can be invoked.

1. Definition of histories

We assume that a physical system $\mathcal{S}$ is described by a density matrix ρ_0 at some initial time t_0 and define a sequence of arbitrary times $t_1 < t_2 < \cdots < t_n$ with $t_1 > t_0$. For each time point t_i in this sequence, we consider an exhaustive set $\mathcal{P}^{(i)} = \{\widehat{P}^{(i)}_{\alpha_i}(t_i) \,|\, \alpha_i = 1 \cdots m_i\}$, $1 \le i \le n$, of mutually orthogonal Hermitian projection operators $\widehat{P}^{(i)}_{\alpha_i}(t_i)$, obeying

$$\sum_{\alpha_i} \widehat{P}^{(i)}_{\alpha_i}(t_i) = 1, \quad \widehat{P}^{(i)}_{\alpha_i}(t_i)\widehat{P}^{(i)}_{\beta_i}(t_i) = \delta_{\alpha_i,\beta_i}\widehat{P}^{(i)}_{\alpha_i}(t_i), \tag{4.3}$$

and evolving, using the Heisenberg picture, according to

$$\widehat{P}^{(i)}_{\alpha_i}(t) = U^\dagger(t_0,t)\widehat{P}^{(i)}_{\alpha_i}(t_0)U(t_0,t), \tag{4.4}$$

where $U(t_0,t)$ is the operator that dynamically propagates the state vector from t_0 to t.

A possible, "maximally fine-grained" history is defined by the sequence of times $t_1 < t_2 < \cdots < t_n$ and by the choice of one projection operator in the set $\mathcal{P}^{(i)}$ for each time point t_i in the sequence, i.e., by the set

$$\mathcal{H}_{\{\alpha\}} = \{\widehat{P}^{(1)}_{\alpha_1}(t_1), \widehat{P}^{(2)}_{\alpha_2}(t_2), \ldots, \widehat{P}^{(n)}_{\alpha_n}(t_n)\}. \tag{4.5}$$

We also define the set $\mathfrak{H} = \{\mathcal{H}_{\{\alpha\}}\}$ of all possible histories for a given time sequence $t_1 < t_2 < \cdots < t_n$. The natural interpretation of a history $\mathcal{H}_{\{\alpha\}}$ is then to take it as a series of propositions of the form "the system $\mathcal{S}$ was, at time t_i, in a state of the subspace spanned by $\widehat{P}^{(i)}_{\alpha_i}(t_i)$."

Maximally fine-grained histories can be combined to form "coarse-grained" sets which assign to each time point t_i a linear combination

$$\widehat{Q}^{(i)}_{\beta_i}(t_i) = \sum_{\alpha_i} \pi^{(i)}_{\alpha_i}\widehat{P}^{(i)}_{\alpha_i}(t_i), \quad \pi^{(i)}_{\alpha_i} \in \{0,1\} \tag{4.6}$$

of the original projection operators $\widehat{P}^{(i)}_{\alpha_i}(t_i)$.

So far, the projection operators $\widehat{P}^{(i)}_{\alpha_i}(t_i)$ chosen at a certain instant t_i in time in order to form a history $\mathcal{H}_{\{\alpha\}}$ were independent of the choice of the projection operators at earlier times $t_0 < t < t_i$ in $\mathcal{H}_{\{\alpha\}}$. This situation was generalized by Omnès (1988a,b,c, 1990, 1992) to include "branch-dependent" histories of the form (see also Gell-Mann and Hartle, 1993)

$$\mathcal{H}_{\{\alpha\}} = \{\widehat{P}^{(1)}_{\alpha_1}(t_1), \widehat{P}^{(2,\alpha_1)}_{\alpha_2}(t_2), \ldots, \widehat{P}^{(n,\alpha_1,\ldots,\alpha_{n-1})}_{\alpha_n}(t_n)\}. \tag{4.7}$$

2. Probabilities and consistency

In standard quantum mechanics, we can always assign probabilities to single events, represented by the eigenstates of some projection operator $\widehat{P}^{(i)}(t)$, via the rule

$$p(i,t) = \mathrm{Tr}\big[\widehat{P}^{(i)\dagger}(t)\rho(t_0)\widehat{P}^{(i)}(t)\big]. \tag{4.8}$$

The natural extension of this formula to the calculation of the probability $p(\mathcal{H}_{\{\alpha\}})$ of a history $\mathcal{H}_{\{\alpha\}}$ is given by

$$p(\mathcal{H}_{\{\alpha\}}) = \mathcal{D}(\alpha,\alpha), \tag{4.9}$$

where the so-called *decoherence functional* $\mathcal{D}(\alpha,\beta)$ is defined by (Gell-Mann and Hartle, 1990)

$$\mathcal{D}(\alpha,\beta) = \mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n}(t_n)\cdots\widehat{P}^{(1)}_{\alpha_1}(t_1)\rho_0\widehat{P}^{(1)}_{\beta_1}(t_1)\cdots\widehat{P}^{(n)}_{\beta_n}(t_n)\big]. \tag{4.10}$$

If we instead work in the Schrödinger picture, the decoherence functional is

$$\mathcal{D}(\alpha,\beta) = \mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n} U(t_{n-1},t_n)\cdots \widehat{P}^{(1)}_{\alpha_1}\rho(t_1)\widehat{P}^{(1)}_{\beta_1}\cdots U^\dagger(t_{n-1},t_n)\widehat{P}^{(n)}_{\beta_n}(t_n)\big]. \tag{4.11}$$

Consider now the coarse-grained history that arises from a combination of the two maximally fine-grained histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$,

$$\mathcal{H}_{\{\alpha\vee\beta\}} = \{\widehat{P}^{(1)}_{\alpha_1}(t_1)+\widehat{P}^{(1)}_{\beta_1}(t_1),\, \widehat{P}^{(2)}_{\alpha_2}(t_2)+\widehat{P}^{(2)}_{\beta_2}(t_2),\, \dots, \widehat{P}^{(n)}_{\alpha_n}(t_n)+\widehat{P}^{(n)}_{\beta_n}(t_n)\}. \tag{4.12}$$

We interpret each combined projection operator $\widehat{P}^{(i)}_{\alpha_i}(t_i)+\widehat{P}^{(i)}_{\beta_i}(t_i)$ as stating that, at time t_i, the system was in the range described by the union of $\widehat{P}^{(i)}_{\alpha_i}(t_i)$ and $\widehat{P}^{(i)}_{\beta_i}(t_i)$. Accordingly, we would like to require that the probability for a history containing such a combined projection operator be equivalently calculable from the sum of the probabilities of the two histories containing the individual projectors $\widehat{P}^{(i)}_{\alpha_i}(t_i)$ and $\widehat{P}^{(i)}_{\beta_i}(t_i)$, that is,

$$\begin{aligned}
&\mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n}(t_n)\cdots\big(\widehat{P}^{(i)}_{\alpha_i}(t_i)+\widehat{P}^{(i)}_{\beta_i}(t_i)\big)\cdots\widehat{P}^{(1)}_{\alpha_1}(t_1)\rho_0\widehat{P}^{(1)}_{\alpha_1}(t_1)\cdots\big(\widehat{P}^{(i)}_{\alpha_i}(t_i)+\widehat{P}^{(i)}_{\beta_i}(t_i)\big)\cdots\widehat{P}^{(n)}_{\alpha_n}(t_n)\big]\\
&\quad\stackrel{!}{=} \mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n}(t_n)\cdots\widehat{P}^{(i)}_{\alpha_i}(t_i)\cdots\widehat{P}^{(1)}_{\alpha_1}(t_1)\rho_0\widehat{P}^{(1)}_{\alpha_1}(t_1)\cdots\widehat{P}^{(i)}_{\alpha_i}(t_i)\cdots\widehat{P}^{(n)}_{\alpha_n}(t_n)\big]\\
&\qquad+\mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n}(t_n)\cdots\widehat{P}^{(i)}_{\beta_i}(t_i)\cdots\widehat{P}^{(1)}_{\alpha_1}(t_1)\rho_0\widehat{P}^{(1)}_{\alpha_1}(t_1)\cdots\widehat{P}^{(i)}_{\beta_i}(t_i)\cdots\widehat{P}^{(n)}_{\alpha_n}(t_n)\big].
\end{aligned}$$

It can be easily shown that this relation holds if and only if

$$\mathrm{Re}\big\{\mathrm{Tr}\big[\widehat{P}^{(n)}_{\alpha_n}(t_n)\cdots\widehat{P}^{(i)}_{\alpha_i}(t_i)\cdots\widehat{P}^{(1)}_{\alpha_1}(t_1)\rho_0\widehat{P}^{(1)}_{\alpha_1}(t_1)\cdots\widehat{P}^{(i)}_{\beta_i}(t_i)\cdots\widehat{P}^{(n)}_{\alpha_n}(t_n)\big]\big\}=0 \quad \text{if } \alpha_i\neq\beta_i. \tag{4.13}$$

Generalizing this two-projector case to the coarse-grained history $\mathcal{H}_{\{\alpha\vee\beta\}}$ of Eq. (4.12), we arrive at the (sufficient and necessary) *consistency condition* for two histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$ (Griffiths, 1984; Omnès, 1990, 1992),

$$\mathrm{Re}[\mathcal{D}(\alpha,\beta)] = \delta_{\alpha,\beta}\mathcal{D}(\alpha,\alpha). \tag{4.14}$$

If this relation is violated, the usual sum rule for calculating probabilities does not apply. This situation arises when quantum interference between the two combined histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$ is present. Therefore, to ensure that the standard laws of probability theory also hold for coarse-grained histories, the set $\mathfrak{H}$ of possible histories must be consistent in the above sense.

However, Gell-Mann and Hartle (1990) have pointed out that when decoherence effects are included to model the emergence of classicality, it is more natural to require

$$\mathcal{D}(\alpha,\beta) = \delta_{\alpha,\beta}\mathcal{D}(\alpha,\alpha). \tag{4.15}$$

Condition (4.14) has often been referred to as *weak decoherence*, and Eq. (4.15) as *medium decoherence* (for a proposal of a criterion for *strong decoherence*, see Gell-Mann and Hartle, 1998). The set $\mathfrak{H}$ of histories is called consistent (or decoherent) when all its members $\mathcal{H}_{\{\alpha\}}$ fulfill the consistency condition, Eqs. (4.14) or (4.15), i.e., when they can be regarded as independent (noninterfering).

3. Selection of histories and classicality

Even when the stronger consistency criterion (4.15) is imposed on the set $\mathfrak{H}$ of possible histories, the number of mutually incompatible consistent histories remains relatively large (d'Espagnat, 1989; Dowker and Kent, 1996). It is not at all clear *a priori* that at least some of these histories should represent any meaningful set of propositions about the world of our observation. Even if a collection of such "meaningful" histories is found, it leaves open the question how to select such histories and which additional criteria would need to be invoked.

Griffith's (1984) original aim in formulating the consistency criterion was only to allow for a consistent description of sequences of events in closed quantum systems without running into logical contradictions.[23] Commonly, however, consistency has also been tied to the emergence of classicality. For example, the consistency criterion corresponds to the demand for the absence of quantum interference—a property of classicality—between two combined histories. It has become clear that most consistent histories are in fact flagrantly nonclassical (Albrecht, 1993; Dowker and Kent, 1995, 1996; Gell-Mann and Hartle, 1990, 1991b; Paz and Zurek, 1993; Zurek, 1993). For in-

[23] However, Goldstein (1998) used a simple example to argue that the consistent-histories approach can lead to contradictions with respect to a combinination of joint probabilities, even if the consistency criterion is imposed; see also the subsequent exchange of letters in the February 1999 issue of *Physics Today*.

stance, when the projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ are chosen to be the time-evolved eigenstates of the initial density matrix $\rho(t_0)$, the consistency condition will automatically be fulfilled, yet the histories composed of these projection operators have been shown to result in highly nonclassical macroscopic superpositions when applied to standard examples such as quantum measurement or Schrödinger's cat. This demonstrates that the consistency condition cannot serve as a sufficient criterion for classicality.

4. Consistent histories of open systems

Various authors have appealed to interaction with the environment and the resulting decoherence effects in defining additional criteria that would select quasiclassical histories and would also lead to a physical motivation for the consistency criterion (see, for example, Albrecht, 1992, 1993; Anastopoulos, 1996; Dowker and Halliwell, 1992; Finkelstein, 1993; Gell-Mann and Hartle, 1990, 1998; Halliwell, 2001; Paz and Zurek, 1993; Twamley, 1993b; Zurek, 1993). This approach intrinsically requires the notion of local, open systems and the split of the universe into subsystems, in contrast to the original aim of the consistent-histories approach to describe the evolution of a single closed, undivided system (typically the entire universe). The decoherence-based studies then assume the usual decomposition of the total Hilbert space $\mathcal{H}$ into a space $\mathcal{H}_\mathcal{S}$, corresponding to the system $\mathcal{S}$, and $\mathcal{H}_\mathcal{E}$ of an environment $\mathcal{E}$. One then describes the histories of the system $\mathcal{S}$ by employing projection operators that act only on the system, i.e., that are of the form $\widehat{P}_{\alpha_i}^{(i)}(t_i) \otimes \widehat{I}_\mathcal{E}$, where $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ acts only on $\mathcal{H}_\mathcal{S}$ and $\widehat{I}_\mathcal{E}$ is the identity operator in $\mathcal{H}_\mathcal{E}$.

This raises the question of when, i.e., under which circumstances, the reduced density matrix $\rho_\mathcal{S} = \text{Tr}_\mathcal{E}\rho_{\mathcal{S}\mathcal{E}}$ of the system $\mathcal{S}$ suffices to calculate the decoherence functional. The reduced density matrix arises from a nonunitary trace over $\mathcal{E}$ at every time point t_i, whereas the decoherence functional of Eq. (4.11) employs the full, unitarily evolving density matrix $\rho_{\mathcal{S}\mathcal{E}}$ for all times $t_i < t_f$ and only applies a nonunitary trace operation (over both $\mathcal{S}$ and $\mathcal{E}$) at the final time t_f. Paz and Zurek (1993) have answered this (rather technical) question by showing that the decoherence functional can be expressed entirely in terms of the reduced density matrix if the time evolution of the reduced density matrix is independent of the correlations dynamically formed between the system and the environment. A necessary (but not always sufficient) condition for this requirement to be satisfied is given by demanding that the reduced dynamics be governed by a master equation that is local in time.

When a "reduced" decoherence functional exists, at least to a good approximation, i.e., when the reduced dynamics are sufficiently insensitive to the formation of system-environment correlations, the consistency of whole-universe histories, described by a unitarily evolving density matrix $\rho_{\mathcal{S}\mathcal{E}}$ and sequences of projection operators of the form $\widehat{P}_{\alpha_i}^{(i)}(t_i) \otimes \widehat{I}_\mathcal{E}$, will be directly related to that of open-system histories, represented by a nonunitarily evolving reduced density matrix $\rho_\mathcal{S}(t_i)$ and "reduced" projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ (Zurek, 1993).

5. Schmidt states vs pointer basis as projectors

The ability of the instantaneous eigenstates of the reduced density matrix (Schmidt states; see also Sec. III.E.4) to serve as projectors for consistent histories and possibly to lead to the emergence of quasiclassical histories has been studied in much detail (Albrecht, 1992, 1993; Kent and McElwaine, 1997; Paz and Zurek, 1993; Zurek, 1993). Paz and Zurek (1993) have shown that Schmidt projectors $\widehat{P}_{\alpha_i}^{(i)}$, defined by their commutativity with the evolved, path-projected reduced density matrix,

$$\begin{aligned}\big[\widehat{P}_{\alpha_i}^{(i)}, U(t_{i-1},t_i)\{\cdots U(t_1,t_2)\widehat{P}_{\alpha_1}^{(1)}\rho_\mathcal{S}(t_1)\\ \times\ \widehat{P}_{\alpha_1}^{(1)}U^\dagger(t_1,t_2)\cdots\}U^\dagger(t_{i-1},t_i)\big] = 0,\end{aligned} \quad (4.16)$$

will always give rise to an infinite number of sets of consistent histories ("Schmidt histories"). However, these histories are branchdependent [see Eq. (4.7)] and usually extremely unstable, since small modifications of the time sequence used for the projections (for instance by deleting a time point) will typically lead to drastic changes in the resulting history, indicating that Schmidt histories are usually very nonclassical (Paz and Zurek, 1993; Zurek, 1993).

This situation is changed when the time points t_i are chosen such that the intervals $(t_{i+1} - t_i)$ are larger than the typical decoherence time τ_D of the system over which the reduced density matrix becomes approximately diagonal in the preferred pointer basis chosen through environment-induced superselection (see also the discussion in Sec. III.E.4). When the resulting pointer states, rather than the instantaneous Schmidt states, are used to define the projection operators, stable quasiclassical histories will typically emerge (Paz and Zurek, 1993; Zurek, 1993). In this sense, it has been suggested that interaction with the environment can provide the missing selection criterion that ensures the quasiclassicality of histories, i.e., their stability (predictability), and the correspondence of the projection operators (the pointer basis) to the preferred determinate quantities of our experience.

The approximate noninterference, and thus consistency, of histories based on local density operators (energy, number, momentum, charge etc.) as quasiclassical projectors (the so-called *hydrodynamic observables*, see Dowker and Halliwell, 1992; Gell-Mann and Hartle, 1991b; Halliwell, 1998) has been attributed to the dynamical stability exhibited by the eigenstates of the local density operators. This stability leads to decoherence in the corresponding basis (Halliwell, 1998, 1999). It has been argued by Zurek (2003b) that this behavior and thus the special quasiclassical properties of hydrodynamic observables can be explained by the fact that these observables obey the commutativity criterion, Eq. (3.21), of the environment-induced superselection approach.

6. Exact vs approximate consistency

In the idealized case where the pointer states lead to an exact diagonalization of the reduced density matrix, histories composed of the corresponding *pointer projectors* will automatically be consistent. However, under realistic circumstances decoherence will typically lead only to approximate diagonality in the pointer basis. This implies that the consistency criterion will not be fulfilled exactly and that hence the probability sum rules will only hold approximately—although usually, due to the efficiency of decoherence, to a very good approximation (Albrecht, 1992, 1993; Gell-Mann and Hartle, 1991b; Griffiths, 1984; Omnès, 1992, 1994; Paz and Zurek, 1993; Twamley, 1993b; Zurek, 1993). Hence, the consistency criterion has been viewed both as overly restrictive, since the quasiclassical pointer projectors rarely obey the consistency equations exactly, and as insufficient, because it does not give rise to constraints that would single out quasiclassical histories.

A relaxation of the consistency criterion has therefore been suggested, leading to "approximately consistent histories" whose decoherence functional would be allowed to contain nonvanishing off-diagonal terms (corresponding to a violation of the probability sum rules) as long as the net effect of all the off-diagonal terms was "small" in the sense of remaining below the experimentally detectable level (see, for example, Dowker and Halliwell, 1992; Gell-Mann and Hartle, 1991b). Gell-Mann and Hartle (1991b) have even ascribed a fundamental role to such approximately consistent histories, a move that has sparked much controversy and has been considered as unnecessary and irrelevant by some (Dowker and Kent, 1995, 1996). Indeed, if only approximate consistency is demanded, it is difficult to regard this condition as a fundamental concept of a physical theory, and the question of how much consistency is required will inevitably arise.

7. Consistency and environment-induced superselection

The relationship between consistency and environment-induced superselection, and therefore the connection between the decoherence functional and the diagonalization of the reduced density matrix through environmental decoherence, has been investigated by various authors. The basic idea, promoted, for example, by Zurek (1993) and Paz and Zurek (1993), is to suggest that if the interaction with the environment leads to rapid superselection of a preferred basis, which approximately diagonalizes the local density matrix, coarse-grained histories defined in this basis will automatically be (approximately) consistent.

This approach has been explored by Twamley (1993b), who carried out detailed calculations in the context of a quantum optical model of phase-space decoherence and compared the results with two-time projected phase-space histories of the same model system. It was found that when the parameters of the interacting environment were changed such that the degree of diagonality of the reduced density matrix in the emerging preferred pointer basis was increased, histories in that basis also became more consistent. For a similar model, however, Twamley (1993a) also showed that consistency and diagonality in a pointer basis as possible criteria for the emergence of quasiclassicality may exhibit a very different dependence on the initial conditions.

Extensive studies on the connection between Schmidt states, pointer states and consistent quasiclassical histories have also been made by Albrecht (1992, 1993), based on analytical calculations and numerical simulations of toy models for decoherence, including detailed numerical results on the violation of the sum rule for histories composed of different (Schmidt and pointer) projector bases. It was demonstrated that the presence of stable system-environment correlations ("records"), as demanded by the criterion for the selection of the pointer basis, was of great importance in making certain histories consistent. The relevance of "records" for the consistent-histories approach in ensuring the "permanence of the past" has also been emphasized by other authors, for example, by Paz and Zurek (1993) and Zurek (1993, 2003b), and in the "strong decoherence" criterion by Gell-Mann and Hartle (1998). The redundancy with which information about the system is recorded in the environment and can thus be found out by different observers without measurably disturbing the system itself has been suggested to allow for the notion of "objectively existing histories," based on environment-selected projectors that represent sequences of "objectively existing" quasiclassical events (Paz and Zurek, 1993; Zurek, 1993, 2003b, 2004b).

In general, damping of quantum coherence caused by decoherence will necessarily lead to a loss of quantum interference between individual histories (but not vice versa—see the discussion by Twamley, 1993b), since the final trace operation over the environment in the decoherence functional will make the off-diagonal elements very small due to the decoherence-induced approximate mutual orthogonality of the environmental states. Finkelstein (1993) has used this observation to propose a new decoherence condition that coincides with the original definition, Eqs. (4.10) and (4.11), except for restricting the trace to $\mathcal{E}$, rather than tracing over both $\mathcal{S}$ and $\mathcal{E}$. It was shown that this condition not only implies the consistency condition of Eq. (4.15), but also characterizes those histories that decohere due to interaction with the environment and that lead to the formation of "records" of the state of the system in the environment.

8. Summary and discussion

The core difficulty associated with the consistent-histories approach has been to explain the emergence of the classical world of our experience from the underlying quantum nature. Initially, it was hoped that classicality could be derived from the consistency criterion alone. Soon, however, the status and the role of this criterion in the formalism and its proper interpretation became rather unclear and diffuse, since exact consistency was shown to provide neither a necessary nor a sufficient criterion for the selection of quasiclassical histories.

The inclusion of decoherence effects into the consistent histories approach, leading to the emergence of stable quasiclassical pointer states, has been found to yield a highly efficient mechanism and a sensitive criterion for singling out quasiclassical observables that simultaneously fulfill the consistency criterion to a very good approximation due to the suppression of quantum coherence in the state of the system. The central question is then: What is the meaning and the remaining role of an explicit consistency criterion in the light of such "natural" mechanisms for the decoherence of histories? Can one dispose of this criterion as a key element of the fundamental theory by noting that for all "realistic" histories consistency will be likely to arise naturally from environment-induced decoherence alone?

The answer to this question may actually depend on the viewpoint one takes with respect to the aim of the consistent-histories approach. As we have noted before, the original goal was simply to provide a formalism in which one could, in a measurement-free context, assign probabilities to certain sequences of quantum events without logical inconsistencies. The more recent and rather opposite aim would be to provide a formalism that selects only a very small subset of "meaningful" quasiclassical histories, all of which are consistent with our world of experience, and whose projectors can be directly interpreted as objective physical events.

The consideration of decoherence effects that can give rise to effective superselection of possible quasiclassical (and consistent) histories certainly falls into the latter category. It is interesting to note that this approach has also led to a departure from the original "closed systems only" view to the study of local open quantum systems and thus to the decomposition of the total Hilbert space into subsystems, within the consistent-histories formalism. Besides the fact that decoherence intrinsically requires the openness of systems, this move might also reflect the insight that the notion of classicality itself can be viewed as only arising from a conceptual division of the universe into parts (see the discussion in Sec. III.A).

Therefore environment-induced decoherence and superselection have played a remarkable role in consistent-histories interpretations: a practical one by suggesting a physical selection mechanism for quasiclassical histories; and a conceptual one by contributing to a shift in our view of originally rather fundamental concepts, such as consistency, and of the aims of the consistent-histories approach, like the focus on description of closed systems.

V. CONCLUDING REMARKS

We have presented an extensive discussion of the role of decoherence in the foundations of quantum mechanics, with a particular focus on the implications of decoherence for the measurement problem in the context of various interpretations of quantum mechanics.

A key achievement of the decoherence program is the recognition that openness in quantum systems is important for their realistic description. The well-known phenomenon of quantum entanglement had already, early in the history of quantum mechanics, demonstrated that correlations between systems can lead to "paradoxical" properties of the composite system that cannot be composed from the properties of the individual systems. Nonetheless, the viewpoint of classical physics that the idealization of isolated systems is necessary to arrive at an "exact description" of physical systems has influenced quantum theory for a long time. It is the great merit of the decoherence program to have emphasized the ubiquity and essential inescapability of system-environment correlations and to have established the important role of such correlations as factors in the emergence of "classicality" from quantum systems. Decoherence also provides a realistic physical modeling and a generalization of the quantum measurement process, thus enhancing the "black-box" view of measurements in the standard ("orthodox") interpretation and challenging the postulate of fundamentally classical measuring devices in the Copenhagen interpretation.

With respect to the preferred-basis problem of quantum measurement, decoherence provides a very promising definition of preferred pointer states via a physically meaningful requirement, namely, the robustness criterion, and it describes methods for selecting operationally such states, for example, via the commutativity criterion or by extremizing an appropriate measure such as purity or von Neumann entropy. In particular, the fact that macroscopic systems virtually always decohere into position eigenstates gives a physical explanation for why position is the ubiquitous determinate property of the world of our experience.

We have argued that, within the standard interpretation of quantum mechanics, decoherence cannot solve the problem of definite outcomes in quantum measurement: We are still left with a multitude of (albeit individually well-localized quasiclassical) components of the wave function, and we need to supplement or otherwise to interpret this situation in order to explain why and how single outcomes are perceived. Accordingly, we have discussed how environment-induced superselection of quasiclassical pointer states together with the local suppression of interference terms can be put to great use in physically motivating, or potentially disproving, rules and assumptions of alternative interpretive approaches that change (or altogether abandon) the strict orthodox eigenvalue-eigenstate link and/or modify the unitary dynamics to account for the perception of definite outcomes and classicality in general. For example, to name just a few applications, decoherence can provide a universal criterion for the selection of the branches in relative-state interpretations and a physical argument for the noninterference between these branches from the point of view of an observer; in modal interpretations, it can be used to specify empirically adequate sets of properties that can be ascribed to systems; in collapse models, the free parameters (and possibly even the nature of the reduction mechanism itself) might be derivable from environmental interactions; decoherence can also assist in the selection of quasiclassical particle trajectories in Bohmian mechanics, and it can serve as an efficient mechanism for singling out quasiclassical histories in the consistent-histories approach. Moreover, it has become clear that decoherence can ensure the empirical adequacy and thus empirical equivalence of different interpretive approaches, which has led some to the claim that the choice, for example, between the orthodox and the Everett interpretation becomes "purely a matter of taste, roughly equivalent to whether one believes mathematical language or human language to be more fundamental" (Tegmark, 1998, p. 855).

It is fair to say that the decoherence program sheds new light on many foundational aspects of quantum mechanics. It paves a physics-based path towards motivating solutions to the measurement problem; it imposes constraints on the strands of interpretations that seek such a solution and thus makes them also more and more similar to each other. Decoherence remains an ongoing field of intense research, in both the theoretical and experimental domain, and we can expect further implications for the foundations of quantum mechanics from such studies in the near future.

Acknowledgments

The author would like to thank A. Fine for many valuable discussions and comments throughout the process of writing this article. He gratefully acknowledges thoughtful and extensive feedback on this manuscript from S. L. Adler, V. Chaloupka, H.-D. Zeh, and W. H. Zurek.

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