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Nonlinear Dependence in Gold and Silver Futures: Is it Chaos?

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NONLINEAR DEPENDENCE IN GOLD AND SILVER FUTURES: IS IT CHAOS?

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Abstract

We test for the presence of low-dimensional chaotic structure in the gold and silver futures markets. While we find strong evidence of nonlinear dependencies, the evidence is not consistent with chaos. Our test results indicate that ARCH-type processes, with controls for contract-maturity effects, generally explain the nonlinearities in the data. We also make a case that employing seasonally adjusted price series is important to obtaining robust results via some of the existing tests for chaotic structure.

I. Introduction

It has been well documented by natural scientists that nonlinear relationships that are deterministic can yield highly complex time paths that will pass most standard tests of randomness. Such random-looking but deterministic series have been termed chaotic in the literature (see Brock (1986) for a survey). Direct applications of chaos to economic theory has been initiated only in the last twenty years, with researchers employing a range of techniques to test the null of chaos in macroeconomic series (see Baumol and Benhabib (1989) for a review). The evidence of chaos in economic time series such as GNP and unemployment has thus far been weak.

On the other hand, the few studies on the structure of commodity prices, employing a range of statistical tests, have generally found evidence consistent with low dimension chaos: Lichtenberg and Ujihara (1988) apply a nonlinear cobweb model to U.S. crude oil prices; Frank and Stengos (1989) estimate the Correlation Dimension and Kolmogorov entropy for gold and silver spot prices; DeCoster, Labys, and Mitchell (1992) apply Correlation Dimension to daily sugar, silver, copper, and coffee futures prices; Yang and Brorsen (1993) employ Correlation Dimension and the Brock, Dechert, and Scheinkman (BDS) test on several futures markets, including gold and silver.

Why is the evidence of chaos stronger in commodity prices? Nonlinear theorists such as Baumol and Benhabib (1989) have suggested that disaggregated variables (such as commodity prices) that are inherently subject to resource constraints will make better candidates for chaos. Are there other explanations for the differences in the evidence across commodity prices and aggregated economic time series? Most prior studies on the structure of commodity prices suffer from a mixture of short data spans and fairly coarse tests for chaos and have generally failed to control for seasonal variations in commodity prices. To what extent have these factors contributed to the evidence for commodity prices?

In this paper we provide new evidence on the structure of commodity prices while addressing these questions. Our paper, which provides evidence for gold and silver futures prices, is distinguishable from the Frank and Stengos (1989) and/or Yang and Brorsen (1993) studies in that (i) relatively long price histories are examined[1]; (ii) the data are subject to adjustments for seasonalities; (iii) a wide range of ARCH-type models are considered as explanations to the nonlinearities; and (iv) alternate statistical techniques are employed to test the null of chaos. Unlike Frank and Stengos and Yang and Brorsen, we find evidence that is inconsistent with chaos. We make a case that employing seasonally adjusted price series and considering a

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wider range of nonlinear alternatives may be critical to obtaining robust results for chaotic structure.

The next section motivates the tests for chaos and further discusses the implications of chaotic structure in commodity prices. Section III describes the procedures that this paper employs to test the null of chaos. Section IV presents the test results for the two commodities. Section V closes with a summary of the results.

II. Chaos: concepts and implications for commodity markets

As the concepts of chaos are well developed in the literature, our descriptions are brief relative to some papers that we reference here. There are several definitions of chaos in use. A definition similar to the following is commonly found in the literature (for instance, see Brock, Hsieh and LeBaron (1993)): the series a_t has a chaotic explanation if there exists a system (h,F,x_0) where $a_t = h(x_t)$, $x_{t+1} = F(x_t)$, x_0, is the initial condition at $t = 0$, and where h maps the n-dimensional phase space, R^n, to R^1, and F maps R^n to R^n. It is also required that all trajectories, x_t lie on an attractor, A, and nearby trajectories diverge so that the system never reaches an equilibrium or even exactly repeats its path.

Chaotic time paths will have the following properties that should be of special interest to commodity market observers[2]: i) the universality of certain routes that are independent of the details of the map; ii) time paths that are extremely sensitive to microscopic changes in the parameters; this property is often termed sensitive dependence upon initial condition or SDIC[3]; and iii) time series that appear stochastic even though they are generated by deterministic systems; i.e., the empirical spectrum and empirical autocovariance functions of chaotic series are the same as those generated by random variables, implying that chaotic series will not be identified as such by most standard techniques.

The above properties of chaos are commonly demonstrated employing simulated data from the following Logistic equation with a single parameter, w (e.g., Baumol and Benhabib (1989))

$$x_{t+1} = F(x_t) = wx_t(1 - x_t). \qquad (1)$$

A plot of x_{t+1} for, say $w = 3.750$, $x_0 = .10$, would produce a fairly complex time path. Moreover, with only a small change in w, say $w = 3.753$ (an error of .003), the time path will be vastly different after only a few time intervals. Given that measurement of w with infinite accuracy is not practical, both basic forecasting devices—extrapolation and estimation of structural forecasting models—become highly questionable in chaotic systems.

A similar comment may be made with respect to the implications of chaos vis a vis policy makers (market regulators). If the price series is chaotic, it is fair to say that regulators must have some knowledge of F,h to effect meaningful and more-than-transitory changes in the price patterns. Then too, it is not obvious that regulators will succeed in promoting their agenda. Without highly accurate information of F and h, and the current state x_0, chaos would imply that regulators cannot extrapolate past behavior to assess future movements. In effect, they would only be guessing as to the *need* for regulation. In other words, one can make the case that the sensible technical analyst and policy maker ought to be pleased when the concerned nonlinear structure is *not chaotic*.[4]

III. Testing for Chaos

The known tests for chaos try to determine from observed time series data whether h and F are genuinely random. There are three tests that we employ here: the Correlation Dimension of Grassberger and Procaccia (1983), and the BDS statistic of Brock, Deckert, and Scheinkman (1987), and a measure of entropy termed Kolmogorov-Sinai invariant, also known as Kolmogorov entropy. We briefly outline the construction of the tests, but we do not address their properties at length, as they have been well established (for instance, Brock, Hsieh and LeBaron (1993)).

A. Correlation Dimension

Consider the stationary time series x_t, $t = 1 \ldots T$. One imbeds x_t in an m-dimensional space by forming M-histories starting at each date t: $x_t^2 = \{x_t, x_{t+1}\}$, $\ldots$, $x_t^M = \{x_t, x_{t+1}, x_{t+2}, \ldots x_{t+M-1}\}$. One employs the stack of these scalars to carry out the analysis. If the true system is n-dimensional, provided $M \geq 2n + 1$, the M-histories can help recreate the dynamics of the underlying system, if they exist. One can measure the spatial correlations among the M-histories by calculating the correlation integral. For a given embedding dimension M and a distance ϵ, the correlation integral is given by

$$C^M(\epsilon) = \lim_{T\to\infty}\{\text{the number of } (i,j) \text{ for which } \| x_i^M - x_j^M \| \le \epsilon\}T^2 \qquad (2)$$

where $\| \cdot \|$ is the distance induced by the norm. For small values of ϵ, one has $C^M(\epsilon) \sim \epsilon^D$ where D is the dimension of the system (see Grassberger and Procaccia (1983)). A popular approach to approximate the correlation dimension in the face of limited data is to estimate the statistic

$$SC^M = \frac{\{\ln C^M(\epsilon_i) - \ln C^M(\epsilon_{i-1})\}}{\{\ln(\epsilon_i) - \ln(\epsilon_{i-1})\}} \qquad (3)$$

for various levels of M (e.g., Brock and Sayers (1988)). The SC^M statistic is a local estimate of the slope of the C^M versus e function. Following Frank and Stengos (1989), we take the average of the three highest values of SC^M for each: embedding dimension.

There are at least two ways to consider the SC^M estimates. First, the original data may be subjected to shuffling, thus destroying any chaotic structure if it exists. If chaotic, the original series should provide markedly smaller SC^M estimates than their shuffled counterparts (e.g., Scheinkman and LeBaron (1986)). Second, along with the requirement (for chaos) that SC^M stabilizes at some low level as we increase M, we also require that linear transformations of the data leave the dimensionality unchanged (e.g., Brock (1986)). For instance, we would have evidence against chaos if AR errors provide SC^M levels that are dissimilar to that from the original series.

B. BDS Statistic

BDS (1987) employ the correlation integral to obtain a statistical test that has been shown to have strong power in detecting various types of nonlinearity as well as deterministic chaos. BDS show that if x_t is IID with a nondegenerate distribution,

$$C^M(\epsilon) \to C^1(\epsilon)^M, \text{ as } T \to \infty \qquad (4)$$

for fixed M and ϵ. Employing this property, BDS show that the statistic

$$W^M(\epsilon) = \sqrt{T}\,[C^M(\epsilon) - C^1(\epsilon)^M]/\sigma^M(\epsilon) \qquad (5)$$

where σ^M, the standard deviation of $[\cdot]$ has a limiting standard normal distribution under the null hypothesis of IID. W^M is termed the BDS statistic. Nonlinearity will be established if W^M is significant for a stationary series void of linear dependence. The absence of chaos will be suggested if it is demonstrated that the nonlinear structure arises from a known non-deterministic system. For instance, if one obtains significant BDS statistics for a stationary data series, but fails to obtain significant BDS statistics for the standardized residuals from an Auto Regressive Conditional Heteroskedasticity (ARCH) model. It can be said that the ARCH process explains the nonlinearity in the data, precluding low dimension chaos.[5]

C. Kolmogorov Entropy

Kolmogorov entropy quantifies the concept of sensitive dependence on initial conditions. Initially, the two time paths are extremely close so as to be indistinguishable to a casual observer. As time passes, however, the trajectories diverge so that they become distinguishable. Kolmogorov entropy (K) measures the speed with which this takes place. Grassberger and Procaccia (1983) devise a measure for K which is more implementable than earlier measures of entropy. The measure is given by

$$K_2 = \lim_{\epsilon\to 0}\lim_{m\to\infty}\lim_{N\to\infty}\ln\left(\frac{C^M(\epsilon)}{C^{M+1}(\epsilon)}\right). \qquad (6)$$

If a time series is non-complex and completely predictable, $K_2 \to 0$. If the time series is completely random, $K_2 \to \infty$. That is, the lower the value of K_2, the more predictable the system. For chaotic systems, one would expect $0 < K_2 < \infty$, at least in principle.

IV. Evidence from the Gold and Silver Futures Markets

We employ daily prices of the nearby gold and silver futures contracts traded on the Commodity Exchange from January 1975 through June 1995 (5160 observations).[6] We focus our tests on daily returns, which are obtained by taking the relative log of closing prices, or $R_t = \ln(P_t/P_{t-1}) \cdot 100$. We ran several diagnostics for the two return series. Both series are found stationary employing the Augmented Dickey Fuller (ADF) statistics. Both series have linear and nonlinear dependencies as indicated by

Ljung-Box Q(12) statistics on R_t and R_t^2. We also find strong Autoregressive Conditional Heteroskedasticity (ARCH) effects as suggested by ARCH(6) chi-square statistics. Thus, there are clear indications that nonlinear dynamics are generating the gold and silver returns. Whether these dynamics are chaotic in origin is the question that we turn to next.

To eliminate the possibility that the linear structure or seasonalities may be responsible for the rejection of chaos by the tests employed, we first estimate autoregressive models for gold and silver with controls for possible day-of-the-week effects, as in

$$R_t = \sum_{i=1}^{p} \beta_i R_{t-i} + \sum_{j=1}^{5} \gamma_j D_{tj} + \epsilon_t, \qquad (7)$$

where D_{jt} represent day-of-the-week dummy variables. The lag length for each series is selected based on the Akaike criterion. The residual term (ϵ_t) represents the price movements that are purged of linear relationships and seasonal influences. The evidence (available from the authors) suggests a Monday-Effect (negative Monday returns) in both returns akin to that found in world equities. There is also significant linear structure in the returns, up to 4 lags for gold, and 5 lags for silver.[7]

A. Correlation Dimension estimates

Table 1 reports the Correlation Dimension (SC^M) estimates for various components of the gold and silver returns' series alongside that for the Logistic series developed earlier. We report dimension results for embedding up to 20 in order to check for saturation.[8] An absence of saturation provides evidence against chaotic structure. For instance, the SC^M estimates for the Logistic map stay close to 1.00, even as we increase the embedding dimensions. Moreover, the estimates for the Logistic series do not change meaningfully after AR transformation. Thus, as should be expected, the SC^M estimates are not inconsistent with chaos for the Logistic series.

For the gold and silver series the SC^M estimates provide evidence against chaotic structure. If one examines the estimates for the gold returns and AR1 series alone, one could (erroneously) make a case for low dimension chaos: the SC^M statistics seem to 'settle' under 10, and the estimates for the AR(1) series is akin to that for the returns. However, the estimates are substantially higher for the AR(5) and the AR(5) with-seasonal-correction (henceforth [AR(5),S]) series, and not very different from the estimates from the random (gold shuffled) series. Thus, the Correlation Dimension estimates suggest that, after properly taking into account the linear structure and day-of-the-week-

TABLE 1
Correlation Dimension Estimates

The Table reports SC^M statistics for the Logistic series (w = 3.750, n = 2000), daily gold returns, silver returns, and their various components over four embedding dimensions 5, 10, 15, 20. AR(p) represents autoregressive (order p) residuals, AR(p),S represents residuals from autoregressive models that correct for day-of-the-week effects in the data.

M =	5	10	15	20
Logistic	1.02	1.00	1.03	1.06
Logistic AR	0.96	1.06	1.09	1.07
Gold Returns	3.14	5.02	6.36	7.61
Gold AR(1)	3.10	5.45	6.88	8.48
Gold AR(5)	3.18	5.93	7.95	10.58
Gold AR(5),S	3.29	6.08	8.22	11.18
Gold Shuffled	3.30	6.72	9.84	11.49
Silver Returns	3.36	6.06	7.30	10.58
Silver AR(1)	3.70	6.87	8.50	11.36
Silver AR(6)	3.71	6.80	8.62	11.05
Silver AR(6),S	3.75	6.92	9.39	13.05
Silver Shuffled	3.74	6.95	9.82	14.14

effect, there is no chaotic structure in gold prices. In the case of silver, the estimates support a rejection of low dimension chaos for all return components, i.e., R_t, AR(1), AR(6), and the AR(6) with-seasonal-correction (henceforth [AR(6),S]) series.

It is notable that, for both gold and silver, the SC^M estimates for the AR(p) series are generally smaller than that for the [AR(p),S] series. Thus, the Correlation Dimension estimates are found to be sensitive to controls for seasonal effects. This has important implications for future tests for chaos employing SC^M.

B. BDS Test results

Table 2 reports the BDS statistics for [AR(5),S] series, and standardized residuals ($\epsilon/\sqrt{h}$) from the Asymmetric Component Garch model,

$$\textit{Component GARCH: } h_t = q_t + \alpha(\epsilon^2_{t-1} - q_{t-1}) + \beta_1(h_{t-1} - q_{t-1}) + \beta_2 TTM_t$$
$$q_t = \omega + \rho(q_{t-1} - \omega) + \phi(\epsilon^2_{t-1} - h_{t-1}), \qquad (8)$$

where the return equation which provides ϵ_t is the same as in (7), and TTM represents time-to-maturity (in days) of the futures contract.[9] The time to maturity variable is intended to control for any maturity effects in the series (Samuelson (1965)).[10] The BDS statistics are evaluated against critical values obtained by bootstrapping the null distribution for Component GARCH model (critical values for all the GARCH alternatives are available from the authors).

The BDS statistics strongly reject the null of no nonlinearity in the [AR(5),S] errors for both gold and silver futures. This evidence, that the two pre-

TABLE 2

BDS statistics

The figures are BDS statistics for AR(p),S residuals, and standardized residuals $\epsilon/\sqrt{h}$ from Asymmetric Component Garch model. The BDS statistics are evaluated against critical values obtained from Monte Carlo simulations. *** represent the significance level of .01.

Panel A: Gold		M		
ϵ/σ	2	3	4	5
AR(5),S Residuals				
0.50	19.20***	24.76***	29.73***	37.09***
1.00	19.96***	24.61***	27.61***	30.82***
1.50	20.14***	24.43***	27.03***	29.02***
2.00	20.05***	23.79***	26.08***	27.50***
Asymmetric Component GARCH Standard Errors				
0.50	–0.15	0.03	–0.30	–0.14
1.00	–0.32	–0.32	–0.56	–0.46
1.50	–0.35	–0.58	–0.63	–0.46
2.00	–0.24	–0.50	–0.40	–0.17
Panel B: Silver				
AR(6),S Residuals				
0.50	17.25***	21.90***	26.35***	31.44***
1.00	18.02***	21.97***	25.06***	28.13***
1.50	18.22***	22.09***	24.41***	26.09***
2.00	19.04***	22.91***	24.84***	26.01***
Asymmetric Component GARCH Standard Errors				
0.50	–1.92	–1.50	–1.01	–0.75
1.00	–2.35	–1.99	–1.48	–1.03
1.50	–2.26	–2.02	–1.62	–1.35
2.00	–1.38	–1.21	–0.69	–0.42

cious metals have nonlinear dependencies, is consistent with the finding in Frank and Stengos (1989). The BDS statistics for the standardized residuals from the ARCH-type models, however, strongly suggest that the source of the nonlinearity is *not* chaos. For both, the gold and silver contracts, the BDS statistics for the standardized residuals are dramatically lower (relative to those for the [AR(5),S] errors) and consistently insignificant at any reasonable level of confidence. The BDS statistics for the standardized residuals from other ARCH-type models (not reported) were also generally insignificant. On the whole, the BDS test results provide compelling evidence that the nonlinear dependencies in gold and silver prices arise from ARCH-type effects, rather than from a complex, chaotic structure.

C. Entropy estimates

Figure 1 plots the Kolmogorov entropy estimates (embedding dimension 15 to 30) for the Logistic map (w = 3.75, x_0 = .10), [AR(5),S] gold series, [AR(6),S] silver series and the shuffled gold returns. The estimates for the Logistic map and the shuffled series provide the benchmarks for a known chaotic, and a generally random series. The entropy estimates for the [AR(5),S] gold series, [AR(6),S] silver series show little signs of 'settling down' as do those for the Logistic map. They behave much more like the entropy estimates for the shuffled series: a general rise in the K_2 statistic as one increases the embedding dimension. The plot reaffirms the Correlation Dimension and BDS test results: there is no evidence of low dimension chaos in gold and silver futures prices.

V. Conclusion

Employing twenty years of data, we conduct a battery of tests for the presence of low-dimensional chaotic structure in the gold and silver futures prices. Daily returns data from the nearby gold and silver contracts are subjected to Correlation Dimen-

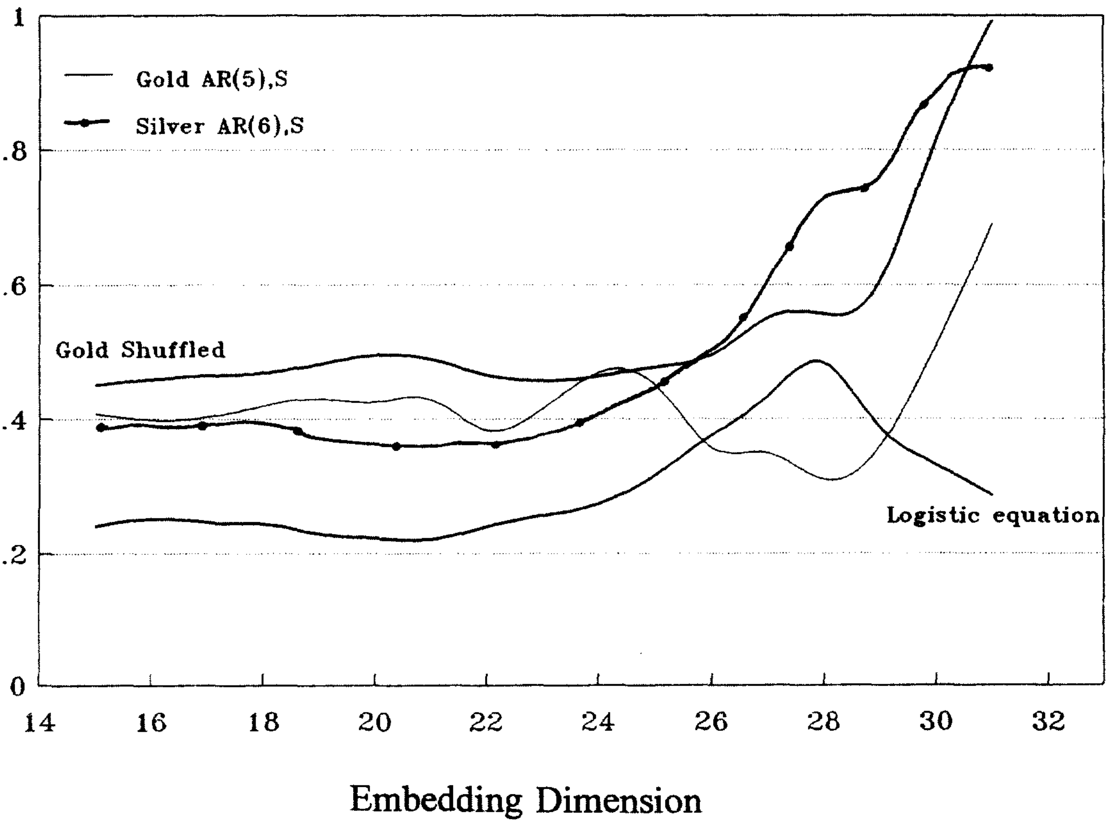

FIGURE 1. Kolmogorov Entropy Estimates

sion tests, BDS tests, and tests for entropy. While we find strong evidence of nonlinear dependence in the data, the evidence is not consistent with chaos. Our test results indicate that ARCH-type processes explain the nonlinearities in the data. We also make a case that employing seasonally adjusted price series is important to obtaining robust results via the existing tests for chaotic structure.

Notes

1. Frank and Stengos study London spot prices for gold and silver over 1/1975–6/1986. Yang and Brorsen study futures prices over 1/1979–12/1988. Our data spans about twice these intervals.
2. See Brock, Hsieh and LeBaron (1993) for a more complete description of the properties.
3. This property follows from the requirement that local trajectories must diverge; if they were to converge, the system would be stable to disturbance, and nonchaotic.
4. It should be noted, however, that chaotic systems may provide some advantage to forecasting/technical analysis in the very-short run. For instance, Clyde and Osler (1997) simulate a chaotic series and demonstrate that that the heads-over-shoulder trading rule will be more consistent at generating profits (relative to random trading) when applied to a known nonlinear system. However, the results also indicate that this consistency declines dramatically, so that the frequency of 'hits' employing the trading rule is not distinguishable from that of a random strategy after just a few trading periods (days).
5. Brock, Hsieh and LeBaron (1993) examine the finite sample distribution of the BDS statistic and find the asymptotic distribution will approximate the distribution of the statistic when the sample is $n > 500$; the embedding dimension is selected to be 5 or lower; and ϵ is selected to be between 0.5 and 2 standard deviations of the data. However, the authors suggest bootstrapping the null distribution to obtain the critical values when applying to standardized residuals from ARCH-type models.
6. The data are obtained from the Futures Industry Institute, Washington, D.C.
7. It should be noted that Frank and Stengos (1989), who find in favor of chaos in gold and silver returns employed residuals that are from an AR1 model with no seasonal correction.
8. Yang and Brorsen (1993), who also calculate Correlation Dimension for gold and silver, compute SC^M only up to $M = 8$.
9. The Asymmetric Component model is a variation of the Threshold Garch model of Rabemananjara and Zakoian (1993). We also estimated other familiar models, Garch in Mean (GARCHM), Garch (1,1) and Exponential Garch (1,1). The standardized residuals from these models were marginally less successful in explaining the nonlinearities in the returns. In the interest of brevity, we only present the results pertaining to the Asymmetric Component Garch model. The BDS results from the alternate ARCH-type models are available from the authors.
10. It is noteworthy that the TTM variable is found to be significant and in support of the Samuelson hypothesis: volatility (conditional variance) rises as one approaches contract maturity.

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