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ON TOTAL PRICE UNCERTAINTY AND THE BEHAVIOR OF A COMPETITIVE FIRM

by Bahram Adrangi* and Kambiz Raffiee**

Abstract

In this paper, a general model of the competitive firm's behavior under output and factor (total) price uncertainty is developed to evaluate the role of market interdependencies in analyzing long-run equilibrium conditions and comparative statics analysis of increased uncertainty in output and input prices. It is demonstrated that the results shown in the literature are a special case of the findings reported here and market interdependencies play a central role in determining the firm's long-run equilibrium under uncertainty.

I. Introduction

It is just over a quarter of a century since the publication of the seminal paper by Sandmo [1971] that formally introduced a systematic formulation of the competitive firm's behavior under output price uncertainty. The theory of the firm under uncertainty has been researched significantly since Sandmo [1971] by examining the firm's operations under various sources of uncertainty in the firm's operations: output price uncertainty, factor price uncertainty, and total (output and factor) price uncertainty. The studies by Chavas and Pope [1985], Demers and Demers [1990], Hartman [1976], Horbulyk [1993], Paris [1989], Pope [1980], and Sandmo [1971] have examined the impact of output price uncertainty; Ormiston and Schlee [1994], the impact of factor cost uncertainty; Booth [1983] and Paris [1988], the impact of total price uncertainty.

These contributions have invariably assumed that the firm's objective is to maximize the shareholder's expected utility function under a given source of price uncertainty and report the comparative statics analysis for a mean-preserving increase in either output price or factor cost uncertainty. None of these studies evaluate the role of market interdependencies in determining a firm's long-run equilibrium conditions under uncertainty.[1]

The present paper will examine the effect of total price uncertainty on the firm's long-run equilibrium where the probability distributions of output and factor prices are not independent. The role of market interdependencies to achieve the firm's equilibrium and the effect of changes in uncertainty on its optimum use of inputs is presented.

The next section describes the basic model of the firm under total price uncertainty and presents the requirements to achieve the long-run competitive equilibrium. Comparative statics results of increased uncertainty in output and input prices are derived in Section III. A brief summary is presented in the final section. The firm's equilibrium under risk neutrality is discussed in the Appendix.

II. The Model

The model explains the firm's long-run behavior which it chooses optimal capital ($\bar{K}$) and labor ($\bar{L}$) to maximize the shareholder's expected utility function. Let $U(\pi)$ be the von Neumann-Morgenstern utility function of the firm with the property that $U'(\pi) > 0$ and $U''(\pi) < 0$ for firms that are risk averse. [2] Hence, the firm's decision problem is to

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$$\max_{K,L} E[U(\pi = pQ - rK - wL)]. \quad (1)$$

where π is profit, Q is output, p, r, and w are the uncertain product price, cost of capital, and wage rate, respectively, representing stochastic random variables with the joint probability distribution function m(p,r,w) defined for p, r, w > 0 with finite moments. The respective expected values for p, r, and w are μ_p, μ_r and μ_w.

The assumptions of an interior solution of the firm's equilibrium to exist are that the firm's production function $Q = Q(K, L)$ is assumed to be strictly concave with factor marginal products strictly positive and increasing at a deceasing rate, i.e., $Q_L > 0$, $Q_K > 0$, $Q_{LL} < 0$, and $Q_{KK} < 0$. Let $E[U(\pi)] \equiv h(K,L)$. Then, the first-order conditions for optimization of (1) are[3]

$$h_K = \frac{\partial E[U(\pi)]}{\partial K} = E[U'(\pi)(pQ_K - r)] = 0, \quad (2)$$

$$h_L = \frac{\partial E[U(\pi)]}{\partial L} = E[U'(\pi)(pQ_L - w)] = 0. \quad (3)$$

Expanding the expectation operator in (2) and (3) gives

$$\mu_p Q_K - \mu_r = \frac{cov[U'(\pi),r]}{E[U'(\pi)]} - Q_K\frac{cov[U'(\pi),p]}{E[U'(\pi)]}, \quad (4)$$

$$\mu_p Q_L - \mu_w = \frac{cov[U'(\pi),w]}{E[U'(\pi)]} - Q_L\frac{cov[U'(\pi),p]}{E[U'(\pi)]}. \quad (5)$$

The firm's equilibrium under uncertainty in (4) and (5) depends on the sign of the covariance terms, $cov[U'(\pi),r]$, $cov[U'(\pi),w]$, and $cov[U'(\pi),p]$. It will be shown below that the signs and the firm's equilibrium under uncertainty depend on: (i) the relationship between the output and factor markets as determined by the properties of the joint probability distribution function of prices and (ii) the firm's attitude toward risk. Market interdependencies have been ignored by the previous studies which focused only on the firm's attitude toward risk. In this paper, market interdependencies are accounted for by examining the joint probability distribution function of wages, capital costs, and output prices.

Let $k(p,w|r) = \frac{m(p,r,w)}{g(r)}$ where $k(p,w|r)$ is the joint conditional probability density function of p and w given r, g(r) is the non-zero marginal probability density function of r, with m(p,r,w) as defined before. Additionally, let $E[U'(\pi)] = \bar{U}'$. Then the covariance term $cov[U'(\pi),r]$, in (4), can be written as

$$cov[U'(\pi),r] = \int_p\int_w\int_r [U'(\pi) - \bar{U}'] \times (r - \mu_r)k(p,w|r)g(r)dpdwdr, \quad (6)$$

or

$$cov[U'(\pi),r] = \int_r [E[U'(\pi)|r] - \bar{U}'](r - \mu_r)g(r)dr. \quad (7)$$

$$\text{where } E[U'(\pi)|r] = \int_p\int_w U'(\pi)k(p,w|r)dpdw.^{4}$$

Since $E[\bar{U}'(r - \mu_r)] = E[U'(\pi)|\mu_r]E(r - \mu_r) = 0$, then (7) can be written as

$$cov[U'(\pi),r] = \int_r [E[U'(\pi)|r] - E[U'(\pi)|\mu_r]](r - \mu_r)g(r)dr. \quad (8)$$

The sign of $cov[U'(\pi),r]$ in (8) depends on the sign of the terms on the right-hand-side integral of the equation because $(r - \mu_r)$ is an increasing function of r. But

$$\frac{\partial[E[U'(\pi)|r] - E[U'(\pi)|\mu_r]]}{\partial r} = \int_p\int_w [-KU''(\pi) + \frac{\partial k}{\partial r} \times \frac{U'(\pi)}{k}]k(p,w|r)dpdw. \quad (9)$$

where $k = k(p,w|r)$. Using the result in (9), the sign of the covariance term in (8) can now be determined as

$$\text{sign}[cov[U'(\pi),r]] = \text{sign}\left[-KU''(\pi) + \frac{\partial k}{\partial r} \times \frac{U'(\pi)}{k}\right]. \quad (10)$$

Similarly, the sign of the remaining covariance terms in (4) and (5) can be shown to be determined as

$$\text{sign}[cov[U'(\pi),w]] = \text{sign}\left[-LU''(\pi) + \frac{\partial f}{\partial w} \times \frac{U'(\pi)}{f}\right], \quad (11)$$

$$\text{sign}[cov[U'(\pi),p]] = \text{sign}\left[Q U''(\pi) + \frac{\partial e}{\partial p} \times \frac{U'(\pi)}{e}\right]. \quad (12)$$

where $f = f(p,r|w)$ and $e = e(r,w|p)$ are the respective joint conditional probability density functions of p and r given w and r and w given p, defined for the non-zero marginal density functions of w and p,t(w) and n(p), as $f(p,r|w) = \frac{m(p,r,w)}{t(w)}$ and $e(r,w|p) = \frac{m(p,r,w)}{n(p)}$.

The assumption that markets are independent implies that $\frac{\partial k(p,w|r)}{\partial r} = 0$, $\frac{\partial f(p,r|w)}{\partial w} = 0$, and $\frac{\partial e(r,w|p)}{\partial p} = 0$. Clearly, these partial derivatives can be equal to zero if and only if $m(p,r,w) = n(p)g(r)t(w)$, that is the output and factor markets are independent.[5] In the special case, when the output and factor markets are independent, and under the assumption that firms are risk averse, the sign of the covariance terms in (10)–(12) can be determined unambiguously as

$$cov[U'(\pi),r] > 0, \quad (13)$$

$$cov[U'(\pi),w] > 0, \quad (14)$$

$$cov[U'(\pi),p] < 0. \quad (15)$$

Using the results in (13)–(15) and in (4) and (5), one gets the well-established condition in the literature that the long-run equilibrium of a risk averse competitive firm is

$$\mu_p Q_K - \mu_r > 0, \quad (16)$$

$$\mu_p Q_L - \mu_w > 0. \quad (17)$$

In other words, under the special case that the markets are independent, the firm's attitude toward risk is sufficient to achieve the long-run equilibrium.

However, if markets are not independent, i.e., $\frac{\partial k(p,w|r)}{\partial r} \neq 0$, $\frac{\partial f(p,r|w)}{\partial w} \neq 0$, and $\frac{\partial e(r,w|p)}{\partial p} \neq 0$, then the firm's attitude toward risk is necessary but not sufficient to have an unambiguous sign on the covariance terms in (10)–(12) and determination of equilibrium under uncertainty in (4) and (5).[6] If output and input markets are interdependent, the long-run equilibrium of a risk averse competitive firm from (4) and (5) is

$$\mu_p Q_K - \mu_r \neq 0, \quad (18)$$

$$\mu_p Q_L - \mu_w \neq 0. \quad (19)$$

Let the optimum capital and labor levels employed by the firm in (16)–(17) and in (18)–(19) be $(\bar{K}^A,\bar{L}^A)$ and $(\bar{K}^B, \bar{L}^B)$, respectively. Clearly, once market interdependencies are taken into consideration, $\bar{K}^A \neq \bar{K}^B$ and $\bar{L}^A \neq \bar{L}^B$. Whether the input levels in (18) and (19) are greater than or less than the input levels in (16) and (17) depends on interdependencies among output and factor markets that determine the sign of covariance terms in (10)–(12) and the resulting equilibrium in (18) and (19).

One can develop scenarios on the structure of interrelationship among markets to examine the behavior of the firm under uncertainty. Consider the possibility that $\frac{\partial k(p,w|r)}{\partial r} > 0$, $\frac{\partial f(p,r|w)}{\partial w} > 0$, and $\frac{\partial e(r,w|p)}{\partial p} < 0$. For risk averse firms, one can then get unambiguous sign on the covariance terms in (10)–(12) resulting in the long-run equilibrium conditions of a risk averse competitive firm are identical to those reported in (16) and (17). This amounts to the conclusion that under uncertainty equilibrium and market interdependencies, the optimal input levels of capital and labor for a risk averse firm would be lower than in certainty equilibrium.

Additionally, the optimum input levels ($\bar{K}^B$ and $\bar{L}^B$) under the special case of interdependent markets, where $\frac{\partial k(p,w|r)}{\partial r} > 0$, $\frac{\partial f(p,r|w)}{\partial w} > 0$, and $\frac{\partial e(r,w|p)}{\partial p} < 0$, can be compared with the optimum input levels ($\bar{K}^A$ and $\bar{L}^A$) under independent markets for a risk averse firm. The results are that the firm employs more of both inputs if markets are interdependent: $\bar{K}^B < \bar{K}^A$ and $\bar{L}^B < \bar{L}^A$. Hence, the interrelationship among markets, established by the conditional probability density functions of output and factor markets, is an important determinant of the competitive firm's long-run equilibrium.

III. Comparative Statics[7]

The role of market interdependencies in deriving the comparative statics results, related with changes in the probability distributions of output and input prices, of a competitive firm is presented in this section. The effects of a marginal increase in price uncertainty are defined by the increased variability of the output and input price density functions in terms of a mean preserving spread. Let us define

$$P^* = \gamma P + \theta_1, \tag{20}$$

$$r^* = \gamma r + \theta_2, \tag{21}$$

$$w^* = \gamma w + \theta_3. \tag{22}$$

where θ_i and γ are the shift parameters which initially equal zero and one, respectively. Then a mean preserving spread for this type of shift in the density functions of P*, r*, and w* leaves their means unchanged, that is

$$dE(P^*) = dE(\gamma P + \theta_1) = \mu_p d\gamma + d\theta_1 = 0, \tag{23}$$

$$dE(r^*) = dE(\gamma r + \theta_2) = \mu_r d\gamma + d\theta_2 = 0, \tag{24}$$

$$dE(w^*) = dE(\gamma w + \theta_3) = \mu_w d\gamma + d\theta_3 = 0. \tag{25}$$

Then (23)–(25) imply

$$\frac{d\theta_1}{d\gamma} = -\mu_p, \tag{26}$$

$$\frac{d\theta_2}{d\gamma} = -\mu_r, \tag{27}$$

$$\frac{d\theta_3}{d\gamma} = -\mu_w. \tag{28}$$

Differentiating the first-order conditions in (2) and (3), evaluated at $\theta_i = 0$ and $\gamma = 1$, and using (26)–(28) yields

$$Q_{KK}\frac{dK}{d\gamma} + Q_{KL}\frac{dL}{d\gamma} = \frac{1}{\Psi}\left[\frac{\partial cov[U'(\pi),r]}{\partial\gamma} - \frac{\partial cov[U'(\pi),p]}{\partial\gamma}Q_K\right], \tag{29}$$

$$Q_{LK}\frac{dK}{d\gamma} + Q_{LL}\frac{dL}{d\gamma} = \frac{1}{\Psi}\left[\frac{\partial cov[U'(\pi),w]}{\partial\gamma} - \frac{\partial cov[U'(\pi),p]}{\partial\gamma}Q_L\right]. \tag{30}$$

where $\Psi = \mu_p E[U'(\pi)] + cov[U'(\pi),p]$.

The partial derivative of the covariance terms in (29) and (30) with respect to γ is

$$\frac{\partial cov[U(\pi),w]}{\partial\gamma} = \int_p\int_r\left[[(p-\mu_p)Q - (r-\mu_r)K - (w-\mu_w)L]U''(\pi) + \frac{\partial f}{\partial\gamma}\times\frac{U'(\pi)}{f}\right]f(p,r|w)dpdr, \tag{31}$$

$$\frac{\partial cov[U'(\pi),r]}{\partial\gamma} = \int_p\int_w\left[[(p-\mu_p)Q - (r-\mu_r)K - (w-\mu_w)L]U''(\pi) + \frac{\partial k}{\partial\gamma}\times\frac{U'(\pi)}{k}\right]k(p,w|r)dpdw, \tag{32}$$

$$\frac{\partial cov[U'(\pi),p]}{\partial\gamma} = \int_r\int_w\left[[(p-\mu_p)Q - (r-\mu_r)K - (w-\mu_w)L]U''(\pi) + \frac{\partial e}{\partial\gamma}\times\frac{U'(\pi)}{e}\right]e(r,w|p)drdw. \tag{33}$$

The covariance terms' sign in (31)–(33), and the subsequent comparative statics results of a mean preserving spread in the output and input price density functions in (29) and (30), are determined by the firm's attitude toward risk, i.e., the sign of $U''(\pi)$, and the interrelationship among markets, $\frac{\partial f}{\partial\gamma}, \frac{\partial k}{\partial\gamma}$, and $\frac{\partial e}{\partial\gamma}$. The firm's attitude toward risk is necessary but not sufficient to have determinate comparative statics results in (29) and (30).

IV. Concluding Comments

The literature on the behavior of a firm under uncertainty has generally overlooked the interdependencies among output and factor markets. Under the special case that markets are independent, the firm's attitude toward risk is sufficient for deriving the long-run equilibrium conditions. It is important to incorporate the interrelationship among markets in examining the firm's behavior under uncertainty. In this paper, a general model of the firm's behavior under output and factor price uncertainty is developed to evaluate the role of market interdependencies in analyzing the long-run equilibrium conditions.

The results show that additional assumptions are necessary to derive the firm's long-run equilibrium under uncertainty. Market interdependencies play a central role in determining the firm's long-run equilibrium rendering previous results, described in the literature, as special cases of conditions reported here. Our findings also demonstrate that the firm's attitude toward risk is necessary but not sufficient to obtain a long-run competitive equilibrium.

Notes

1. However, the analysis in Booth [1983] assumes that the output price and input prices are all drawn from a multivariate normal distribution. Complete independence and perfect correlation among output and input prices are special cases in his treatment.
2. The assumption that firms are risk averse needs further explanation. Since the firm's profit is the only argument included in the profit function, the owners may prefer that managers exchange profit for less risk. If the firm's owners hold a diversified portfolio of assets rather than just this one firm, then they want their managers to be risk neutral and to maximize the firm's expected utility of profit. In other words, departure from the assumption of risk aversion is a real possibility. The recent trend in human resource management is toward performance-based compensation, and prolification of stock options as part of the compensation package [Abowd and Bognano, 1994]. Koretz [1995] finds that, among a group of surveyed firms, performance of the firm was positively correlated with the degree of CEO ownership. This is not surprising as one of the goals of performance-based compensation is to deal with agency problems that existed. Jensen and Meckling [1976] define owners as principals and the manager as owners' agent. If the manager is a utility-maximizing individual, and his personal utility function is influenced by variables other than the owners,' then the manager may not always act in the best interest of the principals. However, in cases that the manager's utility function is affected by the firm's profits, as is the case when compensation is performance-based, then the utility functions of the managers and the principals tend to coincide, at least as far as the firm-related decisions are involved. Therefore, it is safe to assume that firm's managers behave similar to the firm's owners and may become risk averse in their decisions. However, assuming that the firm's owners hold a diversified portfolio of assets, managers, as well as owners, may become risk neutral or risk takers. Assuming this scenario, curvature of the utility function may be altered so as to allow for the possibility that $U''(\pi) = 0$ for risk neutrality or $U''(\pi) > 0$ for risk loving. The results for risk neutrality case are presented in the Appendix.
3. The sufficient second-order conditions for the maximum are that $h_{KK} = \dfrac{\partial^2 E[U(\pi)]}{\partial K^2} < 0$, $h_{LL} = \dfrac{\partial^2 E[U(\pi)]}{\partial L^2} < 0$, and $h_{KK}h_{LL} - h^2_{KL} > 0$.
4 In deriving (7) from (6), the result that $\int_p \int_w k(p,w|r)dpdw = 1$ is used.
5. The assumption that output and input price distributions are independent could roughly be interpreted as prices of inputs and outputs being independently determined. The general equilibrium model of markets shows that input and output prices are determined within the market mechanism. For example, with deregulated markets and rapid transmission of information, transportation costs almost instantaneously adjust to the possible price volatility in the crude oil market, affecting all sectors of the economy. Therefore, relaxing the mutually independence of price distribution assumption leads to a more realistic model that fits today's real-world economy.
6. In other words, market interdependencies amount to having non-zero partial derivative of the conditional densities of output and input prices with respect to a given price. The conditions $\dfrac{\partial k(p,w|r)}{\partial r} \neq 0$, $\dfrac{\partial f(p,r|w)}{\partial w} \neq 0$, and $\dfrac{\partial e(r,w|p)}{\partial p} \neq 0$ imply an increase or decrease in uncertainty to the firm associated with a price change. For example, $\dfrac{\partial k(p,w|r)}{\partial r} > 0$ and $\dfrac{\partial f(p,r|w)}{\partial w} > 0$ imply an increase in uncertainty of capital and labor markets to the firm as a result of a rise in capital and labor prices, respective-

ly. On the other hand, $\frac{\partial e(r,w|p)}{\partial p} < 0$ implies a decrease in uncertainty of product market to the firm for an increase in output price.

7. We thank an anonymous referee for raising the issues and suggesting the references [Hadar and Seo, 1990 and Hadar and Russell, 1974] that motivated us to write this section.

Appendix A

In this appendix, the competitive firm's behavior under total price uncertainty when its managers are risk neutral is examined. With risk neutrality, i.e., $U''(\pi) = 0$, equations (10)–(12) in the paper become

$$\mathrm{sign}[cov[U'(\pi),r]] = \mathrm{sign}\left[\frac{\partial k}{\partial r} \times \frac{U'(\pi)}{k}\right], \quad \text{(A1)}$$

$$\mathrm{sign}[cov[U'(\pi),w]] = \mathrm{sign}\left[\frac{\partial f}{\partial w} \times \frac{U'(\pi)}{f}\right], \quad \text{(A2)}$$

$$\mathrm{sign}[cov[U'(\pi),p]] = \mathrm{sign}\left[\frac{\partial e}{\partial p} \times \frac{U'(\pi)}{e}\right]. \quad \text{(A3)}$$

Following the general model of interdependent markets, the covariance terms in (A1)–(A3) are not equal to zero under risk neutrality and their sign is determined by the interrelationship among markets. Now, the firm's equilibrium under risk neutrality is solely established by market interdependencies. Using equations (2) and (3), the firm's equilibrium under risk neutrality and market interdependencies is

$$\mu_p Q_K - \mu_r \neq 0, \quad \text{(A4)}$$

$$\mu_p Q_L - \mu_w \neq 0. \quad \text{(A5)}$$

Equations (A4) and (A5) under risk neutrality and market independencies become

$$\mu_p Q_K - \mu_r = 0, \quad \text{(A6)}$$

$$\mu_p Q_L - \mu_w = 0. \quad \text{(A7)}$$

Let the optimum capital and labor levels employed by the firm in (A4)–(A5) and in (A6)–(A7) be $(\bar{K}^C, \bar{L}^C)$ and $(\bar{K}^D, \bar{L}^D)$, respectively. Clearly, once market interdependencies are taken into consideration, $\bar{K}^C \neq \bar{K}^D$ and $\bar{L}^C \neq \bar{L}^D$ Whether the input levels in (A4) and (A5) are greater than or less than the input levels in (A6) and (A7) depends on interdependencies among output and factor markets that determine the sign of covariance terms in (A1) to (A3) and the resulting equilibrium in (A5) and (A6). The analysis is similar to the results discussed for a risk averse firm in Section III of the paper.

References

Abowd J.; Bognano, M. "Nice Work." *The Economist,* December 10, 1994, p. 67.

Booth, L. "Total Price Uncertainty and the Theory of the Competitive Firm." *Economica,* May 1983, pp. 183–91.

Chavas, J. P.; Pope, R. "Price Uncertainty and Competitive Firm Behavior." *Journal of Economics and Business,* August 1985, pp. 223–35.

Demers, F.; Demers, M. "Price Uncertainty, the Competitive Firm and the Dual Theory of Choice under Risk." *European Economic Review,* September 1990, 1181–99.

Jensen, M. C.; Meckling, W. H. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics,* October 1976, pp. 305–60.

Hadar, J.; Seo, T. K. "The Effects of Shifts in a Return Distribution on Optimal Portfolios." *International Economic Review,* August 1990, pp. 721–36.

Hadar, J.; Russell, W. R. "Diversification of Prospects." *Journal of Economic Theory,* March 1974, pp. 231–39.

Hartman, R. "Factor Demand with Output Price Uncertainty." *American Economic Review,* September 1976, pp. 675–81.

Horbulyk, T. "On Output Price Uncertainty and the Comparative Statics of Industry Equilibrium." *Southern Economic Journal,* July 1993, pp. 36–48.

Koretz, G. "When the Boss Has a Big Stake." *Business Week,* February 13, 1995, p. 28.

Ormiston, M.; Schlee, E. "Wage Uncertainty and Competitive Equilibrium in Labor Markets." *Economica,* May 1994, pp. 137–45.

Paris, Q. "Long-run Comparative Statics under Output and Land Price Uncertainty." *American Journal of Agricultural Economics,* February 1988, pp. 133–41.

———. "Broken Symmetry, Symmetry, and Price Uncertainty: Short and Long-run Comparative Statics of the Competitive Firm." *European Economic Review,* July 1989, pp. 1227–39.
Pope, R. "The Generalized Envelope Theorem and Price Uncertainty." *International Economic Review,* February 1980, pp. 75–86.
Sandmo, A. "On the Theory of the Competitive Firm under Price Uncertainty." *American Economic Review,* March 1971, pp. 65–73.