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The Effects of Noise Due to Random Undetected Tilts and Paleosecular Variation on Regional Paleomagnetic Directions

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Random tilting of a single paleomagnetic vector produces a distribution of vectors which is not rotationally symmetric about the original vector and therefore not Fisherian. Monte Carlo simulations were performed on two types of vector distributions: (1) distributions of vectors formed by perturbing a single original vector with a Fisher distribution of bedding poles (each defining a tilt correction) and (2) standard Fisher distributions. These simulations demonstrate that inclinations of vectors drawn from both distributions are biased toward shallow inclinations. There is a greater likelihood of statistically "drawing" a vector shallower than the true mean vector than of drawing one that is steeper. The estimated probability increases as a function of angular dispersion and inclination of the true mean vector. Consequently, the interpretation of inclination-only data from either type of distribution is not straightforward, especially when the expected paleolatitude is greater than about 50°. Because of the symmetry of the two distributions, declinations of vectors in each distribution are unbiased. The Fisher mean direction of the distribution of vectors formed by perturbing a single vector with random undetected tilts is biased toward shallow inclinations, but this bias is insignificant for angular dispersions of bedding poles less than 20°. This observation implies that the mean pole calculated from a large set of paleomagnetic directions obtained for coeval rocks over a region will be effectively unbiased by random undetected tilts of those rocks provided the angular dispersion of the undetected tilts is less than about 20°. However, the bias of the mean can be significant for large (>20°) angular dispersion of tilts. The amount of bias of the mean direction maximizes at about 10°–12° in mid-latitude regions but is usually less than 8°. Consequently, large (>12°) inclination discordances are probably not the result of random undetected tilts, even if the angular dispersion of the tilts exceeds 20°.

INTRODUCTION

Applications of paleomagnetism to problems in regional tectonics usually require an estimate of the time-averaged paleomagnetic direction thought to have been produced by a geocentric axial dipole (GAD) field. This direction, however, is usually perturbed by random noise from one or both of two major sources: (1) the paleosecular variation (PSV) of the geomagnetic field and (2) undetected tilts of the rocks from which the paleomagnetic directions have been obtained (e.g., small tilts may be undetected in most volcanic rocks, whereas unknown magnitudes of tilt may be undetected in intrusive rocks). Failure to compensate for these noise sources will yield an incorrect estimate of the GAD direction and, consequently, an incorrect tectonic interpretation. It is critical, then, to examine whether or not the noise produced by these sources is truly random or systematic such as to produce biases in our estimates of the true mean GAD direction.

Several workers have noted that perturbations from PSV seem to produce a *Fisher* [1953] distribution of virtual geomagnetic poles (VGPs) [*Cox*, 1970; *Baag and Helsley*, 1974; *Cox and Gordon*, 1984]. However, *Cox and Gordon* [1984], *McFadden and Reid* [1982], *Kono* [1980], and *Briden and Ward* [1966] have shown that inclinations of vectors drawn from a Fisher distribution are biased toward shallow inclinations such that a mean inclination based on inclination-only data will be shallower than the true mean inclination. Figure 1 illustrates this concept. Likewise, a distribution of virtual geomagnetic colatitudes (VGCs) drawn from a Fisher distribution of VGPs is biased toward larger colatitude values. Several of these workers have developed correction factors for mean inclinations and mean colatitudes as well as dispersion estimates in such cases. However, it is not trivial to extract the actual probability of drawing a single vector with an inclination shallower than the mean direction from a Fisher distributed population of vectors from their work, although the required equations are given. In this paper we estimate the appropriate probabilities and examine their possible significance.

Noise due to random undetected tilts is conceptually equivalent to the detrital remanence acquisition process envisioned by *Griffiths et al.* [1960]. In their model, spherical ferromagnetic grains settle in a column of water with their magnetic moments aligned with the ambient geomagnetic field. This alignment is perturbed upon contact with a generally horizontal but uneven substrate as each grain rolls into the nearest depression. The *Griffiths et al.* [1960] problem of rolling spheres might be formulated more completely as a true direction perturbed by a Fisher distribution of bedding poles, with each bedding pole defining the strike and dip of a plane describing the rotation (or "tilting") of

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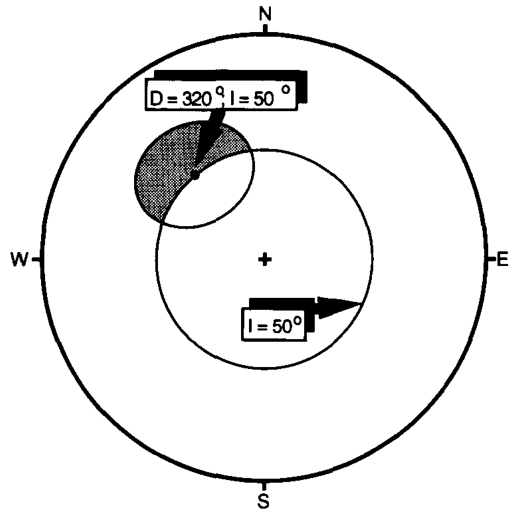

Fig. 1. Equal area projection of a contour of a Fisher distribution about a declination of 320° and an inclination of 50° at 20° radius. Large circle labeled "I = 50°" is the loci of all points having inclination equal to 50°. The stippled area indicates the region within the distribution contour with inclinations shallower than the true mean direction. This area is greater than the corresponding open region with inclinations steeper than the true mean direction.

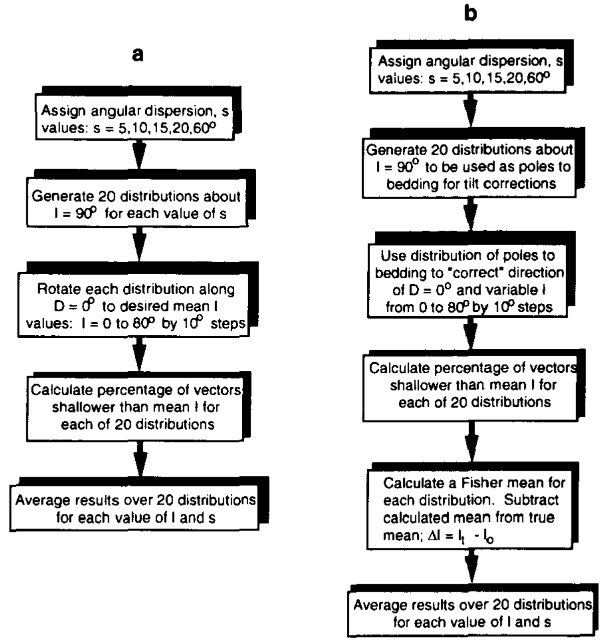

Fig. 3. Flow chart of the procedures used to estimate the probability of drawing an individual shallow inclination from (and biases of the mean directions of) (*a*) Fisher distributions and (*b*) distributions formed by the perturbation of a single direction by a Fisher distribution of bedding poles.

each rolling grain. The resulting distribution is not rotationally symmetric about the mean direction and therefore not Fisherian. Figure 2 illustrates the difference between the two distributions. The Fisher mean direction calculated using both declination and inclination components of the new perturbed distribution will not yield an unbiased estimate of the true mean direction. The calculated Fisher mean direction will be shallower than the true mean direction [*Griffiths et al.*, 1960]. The analysis of *Griffiths et al.* [1960], however, used the simplifying assumption that each grain rolled through a constant angle. In addition, the probabilities of drawing shallow versus steep directions were not calculated.

This paper presents a Monte Carlo simulation of Fisher distributions and distributions resulting from randomly directed undetected tilts of varying magnitude (perturbed distributions) to estimate (1) the probabilities of drawing shallow versus steep directions for single samples from both distributions and (2) the amount and sense of bias of the estimate in the mean direction from a perturbed distribution.

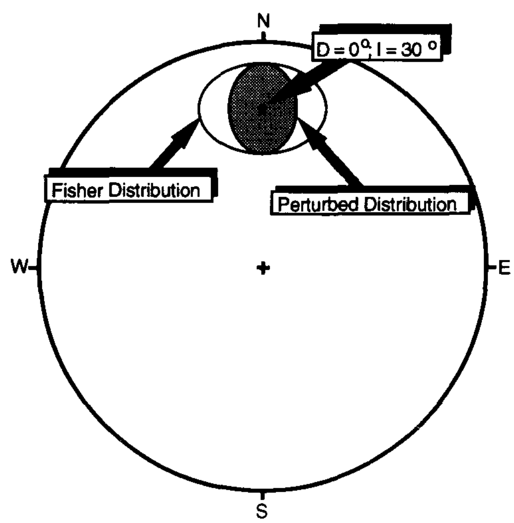

Fig. 2. Equal area projection showing the difference between the *Fisher* [1953] distribution and that formed by perturbation of a single direction by a Fisher distribution of bedding poles. The oval labeled "perturbed distribution" is a contour of a perturbed distribution centered on a declination of 0° and an inclination of 30° using a constant 20° angle of tilt. The oval labeled "Fisher distribution" is a contour of a Fisher distribution about the same direction using a 20° radius.

METHODS AND RESULTS

Figure 3 illustrates the method used to determine the probabilities of finding shallow versus steep paleomagnetic directions for both the Fisher distribution and the distributions created by random undetected tilts. Each synthetic distribution contained $n = 500$ vectors generated using the algorithm of *Fisher et al.* [1987] that utilizes the *C* language random number function. The results for the Fisher distribution are listed in Table 1 and illustrated in Figure 4. The results for the perturbed distribution are listed in Table 1. Several observations bear on the interpretation of paleomagnetic results:

1. In both Fisher and perturbed distributions the likelihood of drawing a single direction shallower than the true direction is always greater than the likelihood of drawing a direction that is steeper than the true direction. (In the special case where the true inclination I_t is zero, the probability of drawing a shallow inclination equals that of drawing a steep inclination.) The probability of drawing a shallow inclination increases with true inclination and angular dispersion (Figure 4).

2. Because of the symmetry of the Fisher and perturbed distributions the probability of drawing a single direction

TABLE 1. Probability of Obtaining a Paleomagnetic Inclination Shallower Than the True Direction as a Function of True Inclination and Angular Dispersion for Fisher and Perturbed Distributions

I_t, deg	$s = 5°$	$s = 10°$	$s = 15°$	$s = 20°$	$s = 60°$
			Fisher Distribution		
0.0	50.0 ± 2.5	49.6 ± 2.5	50.1 ± 2.2	50.3 ± 2.4	50.2 ± 2.2
10.0	50.2 ± 2.5	50.1 ± 2.5	50.8 ± 2.2	51.2 ± 2.4	53.8 ± 2.1
20.0	50.4 ± 2.5	50.7 ± 2.4	51.5 ± 2.2	52.5 ± 2.6	57.6 ± 2.2
30.0	50.5 ± 2.5	51.2 ± 2.5	52.4 ± 2.3	53.3 ± 2.5	61.7 ± 1.7
40.0	50.8 ± 2.5	51.9 ± 2.5	53.4 ± 2.2	54.5 ± 2.4	67.2 ± 2.0
50.0	51.2 ± 2.5	53.0 ± 2.5	54.3 ± 2.1	56.3 ± 2.3	74.1 ± 2.4
60.0	51.7 ± 2.6	54.0 ± 2.5	56.7 ± 2.0	59.2 ± 2.2	81.9 ± 1.9
70.0	52.9 ± 2.7	56.6 ± 2.5	60.7 ± 1.9	65.5 ± 2.2	90.4 ± 1.3
80.0	55.9 ± 3.3	65.1 ± 2.0	74.7 ± 2.1	83.5 ± 3.8	97.0 ± 0.6
90.0	100.0	100.0	100.0	100.0	100.0
			Perturbed Distribution		
0.0	50.0 ± 2.5	49.8 ± 2.5	50.2 ± 2.3	50.2 ± 2.6	50.2 ± 2.1
10.0	50.5 ± 2.2	50.3 ± 2.8	50.8 ± 2.2	51.1 ± 2.4	54.0 ± 2.1
20.0	50.4 ± 2.5	50.5 ± 2.6	51.5 ± 2.3	51.8 ± 2.5	57.3 ± 2.1
30.0	50.5 ± 2.5	51.3 ± 2.6	52.4 ± 2.3	53.3 ± 2.5	62.3 ± 1.9
40.0	50.8 ± 2.5	51.7 ± 2.2	53.4 ± 2.2	54.4 ± 2.3	67.3 ± 1.7
50.0	51.2 ± 2.6	52.8 ± 2.7	54.6 ± 2.0	56.3 ± 2.3	74.8 ± 1.9
60.0	51.9 ± 2.6	54.2 ± 2.5	56.7 ± 2.0	59.4 ± 2.2	81.9 ± 1.9
70.0	52.8 ± 2.7	56.6 ± 2.5	60.4 ± 2.8	65.3 ± 2.0	90.3 ± 1.4
80.0	55.8 ± 3.3	65.1 ± 2.0	74.7 ± 2.2	82.6 ± 1.8	97.3 ± 0.7
90.0	100.0	100.0	100.0	100.0	100.0

Values are in percent. Data are plotted in Figure 4.

clockwise of the mean direction always equals that of drawing a counterclockwise direction.

3. The mean direction of a Fisher distribution is, of course, unbiased. However, the Fisher mean direction calculated for a perturbed distribution is always shallower than the true mean direction except in the case where the true mean inclination is zero. (In this special case a Fisher mean of a perturbed distribution is unbiased.) However, the amount of bias is generally small (<1°) for angular dispersions of bedding poles less than 20° but becomes significant for high angular dispersions (see Table 2 and Figure 5). Because of the symmetry of the perturbed distribution the declination of a Fisher mean direction calculated for a perturbed distribution is unbiased.

TABLE 2. Inclination Error, $\Delta I = I_t - I_o$, for Perturbed Distributions as a Function of True Inclination I_t and Angular Dispersion s of the Fisher Distribution of Bedding Poles Used to Generate the Perturbed Distributions

I_t, deg	$s = 5°$	$s = 10°$	$s = 15°$	$s = 20°$	$s = 60°$	G*
0.0	0.005	0.040	0.005	0.030	0.030	0.000
10.0	0.043	0.090	0.180	0.350	3.300	3.300
20.0	0.058	0.160	0.340	0.650	6.020	6.400
30.0	0.070	0.210	0.370	0.870	9.200	8.900
40.0	0.079	0.230	0.530	0.920	10.800	10.770
50.0	0.082	0.220	0.530	0.980	11.050	11.530
60.0	0.074	0.240	0.480	0.900	11.000	10.890
70.0	0.840	0.140	0.440	0.630	8.840	8.630
80.0	0.018	0.087	0.200	0.380	5.370	4.810

Inclination error values are in degrees. Data are plotted in Figure 5.

*Data are from the *Griffiths et al.* [1960] equation using a constant rolling angle of 60°.

DISCUSSION

Complications in the application of this analysis can arise from two sources. First, all the estimated probabilities listed in Table 1 apply to Fisher distributions of a vector perturbed by a Fisher distribution of bedding poles. But our understanding of paleosecular variation (PSV) is that VGPs are more nearly Fisher distributed about a paleomagnetic pole than geomagnetic field directions at a single observing locality are [*Cox*, 1970; *Baag and Helsley*, 1974; *Cox and Gordon*, 1984]. Therefore a set of directions produced by

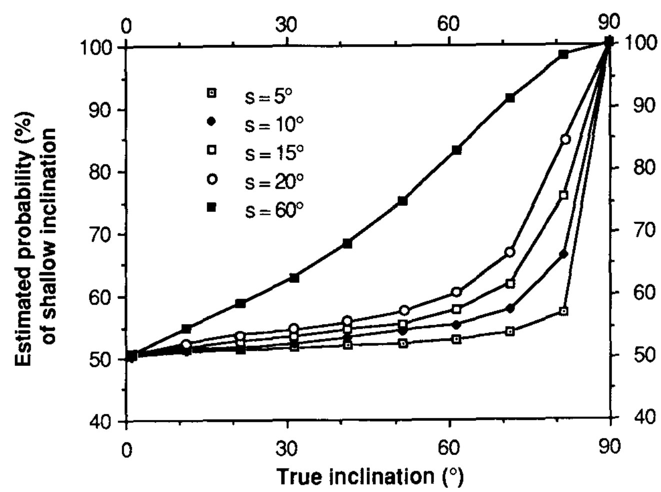

Fig. 4. Estimated probability of drawing a vector with a shallow inclination from a population of Fisher distributed vectors as a function of the true inclination for angular dispersions of 5°, 10°, 15°, 20°, and 60°. The estimated probability of drawing a vector with a shallow inclination from a population of vectors in a perturbed distribution as a function of the true inclination for angular dispersions of bedding poles of 5°, 10°, 15°, 20°, and 60° is statistically indistinguishable from this figure (see Table 1).

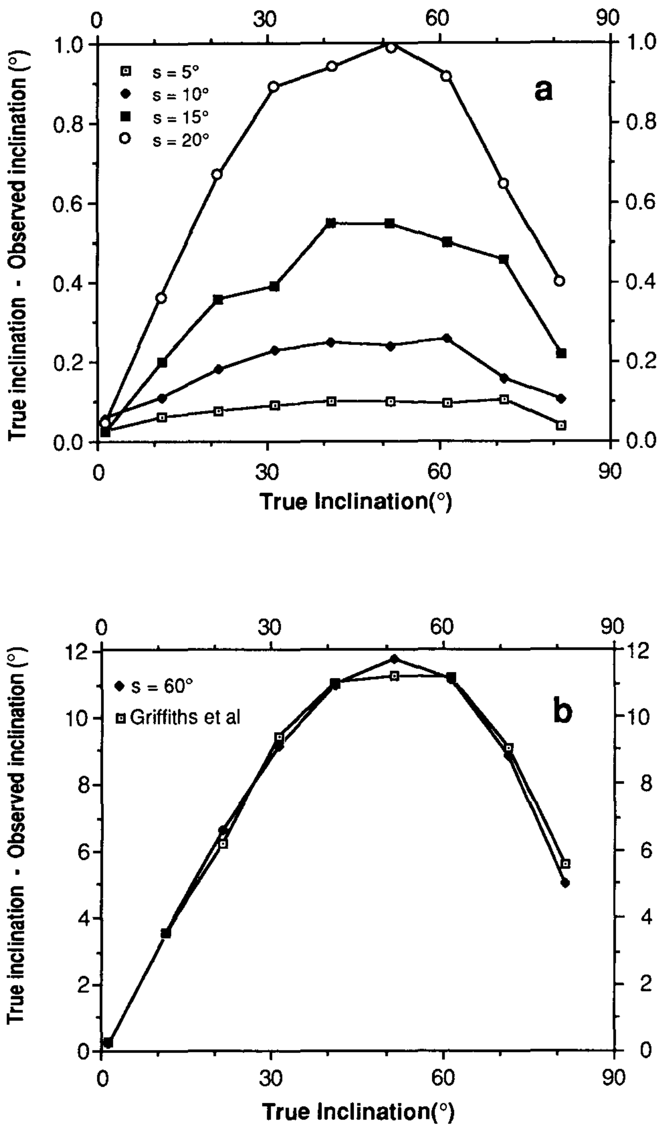

Fig. 5. Inclination error, $\Delta I = I_t - I_o$, as a function of true inclination for perturbed distributions generated from Fisher distributions of bedding poles having initial angular dispersions of (*a*) 5°, 10°, 15°, and 20° and (*b*) inclination error using bedding pole angular dispersion of 60°. Also plotted is a curve generated using the equations of *Griffiths et al.* [1960] with a 60° rolling angle. Note that the actual inclination error is less than 1° in Figure 5*a*.

PSV is expected to be the result of mapping a circular distribution of VGPs into a set of directions (using the dipole formula) (see *Hagstrum et al.* [1987] for further detail).

The problem of evaluating the probability of drawing a single direction from a population of directions produced by PSV being shallower than the true direction can be formulated in the following way [after *Cox and Gordon*, 1984]:

1. Realize that the mathematics of estimating probabilities of shallow directions depends only on the distance of the true mean direction from the vertical (i.e., from $I_t = 90°$). This distance is $90° - I_t$ in direction space.

2. Now visualize the problem of a Fisherian distribution of VGPs in pole space. By viewing directly down on the observing locality (centering the projection on the observing locality), this problem is made analogous to the problem of determining probabilities of shallow directions. The angular distance from the observing locality (equal to the center of projection) to a VGP in the Fisher distribution is the colatitude C of the site with respect to that VGP (equal to the distance from the center of projection). This distance is analogous to the angle $90° - I_t$ in the directional space problem treated above. So the likelihood of observing a colatitude C which is larger than the colatitude of the mean pole (equal to the true colatitude C_t) is analogous to the likelihood of sampling a direction shallower than the true inclination from a Fisherian distribution of directions. Operationally, one can apply the results of Figure 4 by simply replacing the true inclination on the ordinate by $90° - C_t$. The abscissa then becomes the estimated probability of drawing a sample VGP with $C > C_t$.

3. To obtain the estimated probability of sampling a shallow direction from a set of directions resulting from PSV, first convert the true mean inclination I_t to a true mean colatitude C_t. Then determine $90° - C_t$, and use this to locate the point in question on the ordinate of Figure 4. The estimated probability of drawing a sample with $I < I_t$ depends on the angular dispersion of the VGPs and can be read from Figure 4.

In examining the effects of a Fisherian distribution of bedding poles operating on a single direction, we have effectively assumed that a paleomagnetic pole which has completely averaged PSV is perturbed by random undetected tilts. This assumption is rarely practical because the averaging of PSV generally requires sampling of paleomagnetic sites over a large region within which differential tilting may have occurred. A realistic approach requires considering a Fisher distribution of VGPs which is perturbed by a Fisher distribution of bedding poles. In detail this problem becomes very complex. We have performed the calculations for several simulations of this problem, and the estimated probabilities are within a few percent of the estimated probabilities of shallowed inclinations produced by the simple perturbation of a single direction by a Fisherian distribution of bedding poles. However, resulting angular dispersions of VGPs are significantly affected by considering the complete problem. This observation could have implications for analysis of PSV.

CONCLUSIONS

The implications of these results for regional paleomagnetic studies applied to tectonic problems can be divided into two basic classes: (1) paleomagnetic poles calculated from large data sets containing both declination and inclination information and (2) analyses based on single components of magnetizations (i.e., inclination-only data). In addition, we reexamine inclination error in depositional remanent magnetization (DRM) as described by *Griffiths et al.* [1960] in light of the more realistic analysis.

Paleomagnetic Poles

For angular dispersion of random undetected tilts less than about 20°, the mean paleomagnetic pole is biased by no more than 1°. In regional studies of coeval rocks [e.g., *Calderone et al.*, 1990; *Hagstrum et al.*, 1987] the mean poles are effectively unbiased even though tilt corrections in these rocks are not precisely known. For layered rocks, undetected tilts of a magnitude sufficient to produce a significant shallowing of inclination are unlikely. Thus undetected tilts affecting layered rocks are not a viable explanation of shallowing of paleomagnetic directions in these rocks. In rocks lacking paleohorizontal indicators (e.g., plutonic rocks), tilting remains a potent source of deflected paleomagnetic directions.

Single-Component Data

Single vectors drawn from either Fisher distributions or distributions formed by the perturbation of bedding poles tend to be shallower than true mean directions. Consequently, when using VGC-only data drawn from a Fisher distribution of VGPs, it is critical to correct the mean colatitudes using the methods of *Cox and Gordon* [1984] or *McFadden and Reid* [1982]. Since the probability of far-sided colatitude increases with increasing paleolatitude, it is very important to consider whether or not a mean pole is based on a sufficiently large data set such that PSV and random undetected tilting are adequately sampled. The correction factors for the distribution formed by perturbation of a pole or another distribution of poles by a Fisher distribution of bedding poles have not been derived to our knowledge. However, given the similarity of Fisher and perturbed distributions at low angular dispersions, it seems reasonable that the application of the *Cox and Gordon* [1984] or *McFadden and Reid* [1982] corrections to distributions resulting from random undetected tilts would yield a good approximation for angular dispersions less than 20°.

Inclination Error in DRM

Griffiths et al. [1960], on the basis of a simple analysis, concluded that one source of inclination error in sediments was the randomly directed rolling of spherical grains upon contact with a horizontal but uneven substrate. Our analysis supports the conclusion of *Griffiths et al.* [1960] (see Figure 5). However, our analysis shows that in order to produce a significant inclination error, the average angle through which each grain rotates must be large (>20°). *King* [1955, Figure 8] argued that for equant spheres, the maximum rolling angle was slightly greater than 60°. If a 60° rolling angle is maximum and the rolling angles are normally distributed, then the average rolling angle is about one third of the maximum angle, or 20°. With 20° of angular dispersion the maximum inclination error that can be produced by rolling equant spheres is less than 1° (Figure 5), in agreement with the analyses of *King* [1955] and *King and Rees* [1966]. This analysis supersedes the earlier discussion of *Calderone* [1988] and *Calderone and Butler* [1988].

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