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Comment on: "Charge density on a thin straight wire, revisited," by J. D. Jackson

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muonic system in which the muon spends considerable time within the nucleus. The energy value is adjusted upward until the right-hand boundary condition is satisfied (see Fig. 1). The large difference between the seed value and the final energy value (-18.916 vs -10.413 MeV for the $1s$ state) demonstrates the significance of the finite size of the nucleus for muonic atoms.

The fine structure corrections outlined by Tiburzi and Holstein [their Eqs. (16)–(18)] can be easily added to the MATHCAD worksheet (omitted here for the sake of brevity), using traditional numerical algorithms, so that a thorough comparison of theory and experiment can be made.

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Comment on ''Charge density on a thin straight wire, revisited,'' by J. D. Jackson [Am. J. Phys. 68 (9), 789–799 (2000)]

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Jackson's paper[1] supports an emerging consensus[2] that the linear charge density on a conducting wire is uniform, in the zero radius limit. This is easily proved for the special case of an ellipsoid, but Jackson demonstrates that it holds regardless of shape. This conclusion is so counterintuitive that we decided to reexamine the original numerical studies,[3] based on discrete charge distributions, that appeared to confirm the more plausible hypothesis that the charge accumulates preferentially near the ends.

We place N charges at equal spacing on the interval $0<x\leqslant 1$:

$$\begin{aligned} &q_1 \quad \text{at} \quad x_1=1/N,\\ &q_2 \quad \text{at} \quad x_2=2/N,\\ &\cdots\\ &q_n \quad \text{at} \quad x_n=n/N,\\ &\cdots\\ &q_N \quad \text{at} \quad x_N=1 \end{aligned} \tag{1}$$

(and equal charges at the corresponding points on $-1\leqslant x<0$), together with a single charge q_0 at $x_0=0$. We then adjust the charges so that the Coulomb force on each of them except q_N (which is subject to an extra confining force) is zero:

$$\sum_{j=1}^{N}\frac{q_j}{(n+j)^2}+\frac{q_0}{n^2}+\sum_{j=1}^{n-1}\frac{q_j}{(n-j)^2}-\sum_{j=n+1}^{N}\frac{q_j}{(j-n)^2}=0 \quad (n=1,2,...,N-1), \tag{2}$$

subject to the constraint

$$q_0+2\sum_{n=1}^{N}q_n=1 \tag{3}$$

(the scaled total charge on the wire). This does not determine the charge at the center—the force on q_0 is automatically zero, by symmetry. To ensure continuity we choose $q_0=q_1$. What remains is a set of N linear equations for the N unknown charges.

Griffiths and Li[2] solved this system numerically for N up to 100, and persuaded themselves that the linear charge density was approaching a nontrivial limiting form—fairly flat in the center, but with spikes at the ends ($x=\pm 1$). They were seduced by extraordinarily slow convergence as N

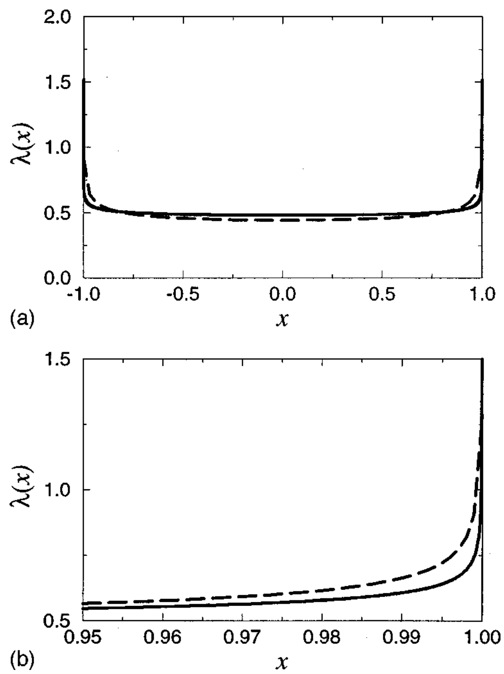

Fig. 1. Linear charge density on a needle, as a function of position. The calculation was done using $2N+1$ point charges equally spaced on the interval from -1 to $+1$, and requiring that the net force on each charge (except the end two) vanish. The total charge on the needle is 1. (a) Solid line: $N=16\,384$; dashed line: $N=32$. (b) Expanded view of the right end; this time the dashed line is $N=1024$.

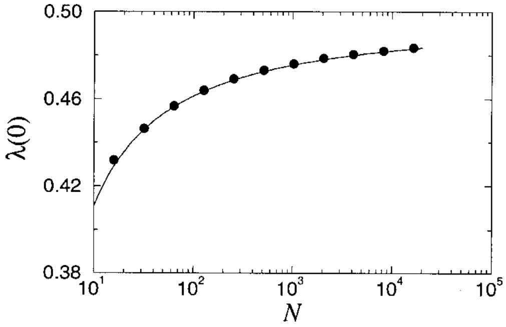

Fig. 2. Charge density at the center of the needle, as a function of N. Dots represent the numerical results. The solid line is the best fit of the form $\lambda(0)=P_1+P_2/\ln N+P_3/(\ln N)^2$ (for N ranging from 32 to 16 384), which occurs for $P_1=0.500$, $P_2=-0.152$, and $P_3=-0.123$. Evidently $\lambda(0)$ approaches the uniform density value of 0.5, as N increases.

$\to\infty$, as we can see from Fig. 1, which extends the calculation out to $N=16\,384$: As N increases, the charge density approaches 1/2, except at the very ends, which occupy a decreasing portion of the length and contain a diminishing fraction of the total charge.

In Fig. 2 we plot the charge density at the center ($\lambda(0)=Nq_0$) as a function of N, to demonstrate the (painfully slow) approach to 0.5. Jackson shows that the natural expansion parameter is Λ^{-1}, where $\Lambda\equiv\ln(4c^2/a^2)$, with $2c$ the length of the wire and a its characteristic ''radius,'' and he suggests that for the discrete model this translates to $\Lambda\sim 2\ln N$. In Fig. 2 the solid line is a best fit of the form

$$\lambda(0)=P_1+\frac{P_2}{\ln N}+\frac{P_3}{(\ln N)^2}; \tag{4}$$

for our data (with N ranging from 32 to 16 384) $P_1=0.500$, $P_2=-0.152$, and $P_3=-0.123$.

In Fig. 3 we plot the charge density at the ends of the wire ($\lambda(1)=Nq_N$), as a function of N. It seems clear that this quantity increases without limit—in fact, our data are well represented by the functional form

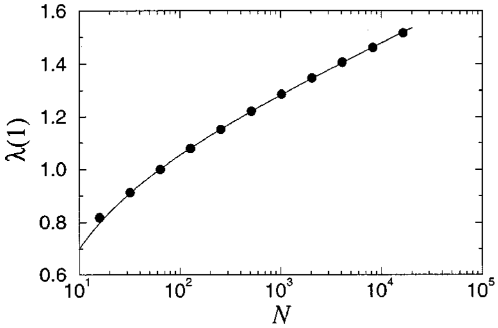

Fig. 3. Charge density at the ends of the needle ($x=\pm 1$), as a function of N. Dots represent the numerical results. The solid line is the best fit of the form $\lambda(1)=[Q_1+Q_2/\ln N+Q_3/(\ln N)^2]\ln N$, which occurs for $Q_1=0.0719$, $Q_2=0.912$, and $Q_3=-0.874$. Evidently $\lambda(\pm 1)$ diverges as N increases.

$$\lambda(1)=\left[Q_1+\frac{Q_2}{\ln N}+\frac{Q_3}{(\ln N)^2}\right]\ln N, \tag{5}$$

with $Q_1=0.0719$, $Q_2=0.912$, and $Q_3=-0.874$ (solid line). Nevertheless, these ''rabbit ears'' in $\lambda(x)$ are of decreasing significance as $N\to\infty$, in the sense that they occupy a diminishing portion of the total length and contain a smaller and smaller fraction of the total charge.

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PREPARATION?

Gibbs began his lectures on thermodynamics with the Carnot cycle, which he always got wrong. After getting thoroughly mixed up he concluded the first lecture with an apology, and in the second lecture he gave it letter perfect. It was in this way he introduced entropy, rather than in the formal way in the ''Heterogeneous Substances''.

E. B. Wilson, a student of J. Willard Gibbs, as quoted by Clifford Truesdell in J. Serrin (editor), *New Perspectives in Thermodynamics* (Springer, New York, 1986), p. 107.